

Surface contribution to the electric dipole moment near an interface, and its effect on power emission

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When a small particle is located near an interface, its electric dipole moment can be induced by laser irradiation. Since the laser light reflects at the interface, this leads to an interference pattern, and the dipole moment is determined by this total field. In addition, the dipole radiation emitted by the particle reflects at the interface, and this field adds to the external field. In this fashion, the dipole moment is altered by the field it emits, and as such the emitted radiation modifies its own source. We have derived a simple expression to take this back action into account. We introduce two resolvent functions, $\Upsilon_{\parallel}(h)$ and $\Upsilon_{\perp}(h)$, which depend on the dimensionless distance h between the particle and the interface. These functions exhibit resonance features due to the underlying back-action mechanism. It is shown that two, one, or no resonance peaks appear in the induced dipole moment. Whether these peaks are present depends on the parameters under consideration. The power emitted by the particle depends on h due to interference between the source radiation and the reflected radiation. With the surface induced contribution to the dipole moment included, an additional h dependence appears. This dependence shows the resonance peaks, which may be amenable to experimental observation. © 2021 Optical Society of America

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1. INTRODUCTION

The power emitted by an oscillating electric dipole moment depends not only on the state of oscillation of the particle emitting the radiation but also on the environment of the emitter. It has long been recognized that a nearby interface modifies the rate of energy emission of the oscillating dipole moment. It has been predicted theoretically [1–10] and confirmed experimentally [11–19] that the emitted power depends on the distance between the particle and the interface. In more recent experiments, nanoparticles are deposited on composite nano-structured substrates, which opens the door to the design of systems with unusual reflection spectra [20]. Emitted dipole radiation (the source field) reflects off the interface, and interference between both fields leads to a modification of the emission rate. Usually, the dipole moment is set in oscillation through irradiation by a laser. If the laser has angular frequency ω , the induced dipole moment has the form

$$\mathbf{d}(t) = \text{Re}(\mathbf{d}e^{-i\omega t}), \quad (1)$$

with $\mathbf{d}$ the complex amplitude. In most theoretical investigations, the value of $\mathbf{d}$ is assumed to be known and determined by independent means. For instance, if the laser is linearly polarized, then $\mathbf{d}$ is real and directed along the polarization direction, and $\mathbf{d}(t)$ oscillates back and forth along the same direction.

In general, laser light will reflect at the surface, and this reflected field adds to the incident field, giving an interference pattern. Then the dipole moment is determined by the total electric field at the location of the dipole.

Interference between the source light and the reflected source light alters the emission rate, but this mechanism does not directly affect the value of $\mathbf{d}$. From a more general point of view, the dipole moment $\mathbf{d}$ is induced by the total external electric field at the location $\mathbf{r}_0$ of the particle as

$$\mathbf{d} = \alpha \mathbf{E}_{\text{ext}}(\mathbf{r}_0), \quad (2)$$

with α the polarizability of the particle (assumed to be a scalar). If the external field is assumed to be the superposition of the laser field and its reflection at the surface, then this determines $\mathbf{d}$, and so $\mathbf{d}$ can be considered a given for the problem. However, the source field also reflects at the interface and adds to the external field at the location of the particle. This source field is emitted by the oscillating $\mathbf{d}(t)$, so $\mathbf{d}$, as given by Eq. (2), is altered by the radiation it produces. This altered $\mathbf{d}$ changes the emission by $\mathbf{d}(t)$, and this changes the reflected field, which then alters $\mathbf{d}$ again, and so on. In this fashion, the value of $\mathbf{d}$ becomes entangled with the emission process and cannot be determined independently anymore.

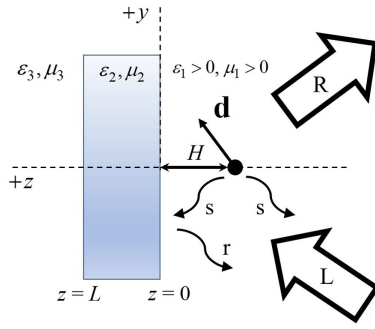


Fig. 1. The figure shows schematically the setup under consideration. The direction of the dipole vector $\mathbf{d}$ is for illustration only, since in general this vector will be complex-valued.

This self-action on $\mathbf{d}$ has been implemented in several numerical studies of scattering by an object near an interface [9,21–24], and its general concept is presented in [25] (Section 15.3). An attempt to compute $\mathbf{d}$ for a dipole near a single interface is given in [21]. The method is based on image theory and leads to a gigantic expression for an involved Green's function. This function is a sum of two integrals, and each contains an infinite series of Bessel functions in the integrand. We shall derive a very simple expression for $\mathbf{d}$, based on the angular spectrum approach.

2. SETUP

The details of the setup are illustrated in Fig. 1. A dipole is embedded in a medium with (relative) permittivity ε_1 and (relative) permeability μ_1 , and both are assumed to be positive. The dipole is located on the z axis, a distance H away from an interface with a material medium. This medium can be a semi-infinite half-space with parameters ε_2 and μ_2 , which may be complex. Or, it can be a layered structure, as in the figure. The only requirement is that the reflection of a plane wave can be accounted for by Fresnel reflection coefficients for s and p polarization. The positive z axis is taken as shown in the figure, and the interface is the xy plane. The incident laser beam is indicated by L , and its reflection at the interface by R . The source radiation (s) from the dipole is emitted in all directions and is partially reflected at the interface (r).

3. LASER AND ITS REFLECTION

The angle of incidence of the laser is θ_i , and we take the plane of incidence as the yz plane. The complex amplitude of the electric field is

$$\mathbf{E}_L(\mathbf{r}) = E_0 \mathbf{e}_L e^{i\mathbf{k}_L \cdot \mathbf{r}}, \quad (3)$$

with $E_0 > 0$ the amplitude, $\mathbf{e}_L$ the complex-valued polarization vector, normalized as $\mathbf{e}_L^* \cdot \mathbf{e}_L = 1$, and $\mathbf{k}_L$ the wave vector. It has to hold that $\mathbf{e}_L \cdot \mathbf{k}_L = 0$, $\mathbf{k}_L \cdot \mathbf{k}_L = n_1^2 k_0^2$, with $n_1 = \sqrt{\varepsilon_1 \mu_1}$ the index of refraction of the embedding medium, and $k_0 = \omega/c$ as the wavenumber in free space. The wave vector is explicitly $\mathbf{k}_L = n_1 k_0 (\mathbf{e}_y \sin \theta_i + \mathbf{e}_z \cos \theta_i)$. To find the reflected field, we need to split the incident field in s and p waves. As phase convention for the polarization vectors, we take $\mathbf{e}_s = -\mathbf{e}_x$ and $\mathbf{e}_p = -\mathbf{e}_y \cos \theta_i + \mathbf{e}_z \sin \theta_i$. Then we expand $\mathbf{e}_L$ as

$$\mathbf{e}_L = N_L \sum_{\sigma=s,p} a_\sigma \mathbf{e}_\sigma, \quad (4)$$

with

$$N_L = \frac{1}{\sqrt{|a_s|^2 + |a_p|^2}}, \quad (5)$$

the normalization constant. The constants a_s and a_p are complex, in general, and these are the free polarization parameters. For instance, for $a_s = 1$, $a_p = 0$ the laser is linearly polarized in the x direction and for $a_s = 1$, $a_p = i$ the laser is left-circularly polarized.

The reflected field is then

$$\mathbf{E}_R(\mathbf{r}) = E_0 N_L \sum_{\sigma=s,p} a_\sigma R_\sigma \mathbf{e}_{\sigma,r} e^{i\mathbf{k}_r \cdot \mathbf{r}}. \quad (6)$$

The polarization vectors are $\mathbf{e}_{s,r} = -\mathbf{e}_x$, $\mathbf{e}_{p,r} = \mathbf{e}_y \cos \theta_i + \mathbf{e}_z \sin \theta_i$, and the wave vector is $\mathbf{k}_r = n_1 k_0 (\mathbf{e}_y \sin \theta_i - \mathbf{e}_z \cos \theta_i)$. Here, R_s and R_p are the Fresnel reflection coefficients for s waves and p waves, respectively. Explicit expressions for a single interface are given in Section 5. We write $\mathbf{E}_{L+R}(\mathbf{r})$ for the sum of the laser field and the reflected field. We only need this field at $\mathbf{r}_0$, the location of the dipole. It then follows that this field can be written as

$$\mathbf{E}_{L+R}(\mathbf{r}_0) = E_0 \mathbf{m}(h), \quad (7)$$

and the mode structure function $\mathbf{m}(h)$ is found to be

$$\begin{aligned} \mathbf{m}(h) = & -N_L a_s \mathbf{e}_x (e^{-iv_1 h} + R_s e^{iv_1 h}) \\ & - N_L a_p \mathbf{e}_y \cos \theta_i (e^{-iv_1 h} - R_p e^{iv_1 h}) \\ & + N_L a_p \mathbf{e}_z \sin \theta_i (e^{-iv_1 h} + R_p e^{iv_1 h}). \end{aligned} \quad (8)$$

Here, $h = k_0 H$ is the dimensionless distance between the dipole and the interface. On this scale, a distance of 2π corresponds to a free-space optical wavelength. We have also introduced the abbreviation $v_1 = n_1 \cos \theta_i$. With Eqs. (4) and (5) the vector $\mathbf{e}_L$ can be constructed, and we can then obtain the alternative form for the mode structure function:

$$\begin{aligned} \mathbf{m}(h) = & \varepsilon_{L,x} \mathbf{e}_x (e^{-iv_1 h} + R_s e^{iv_1 h}) \\ & + \varepsilon_{L,y} \mathbf{e}_y (e^{-iv_1 h} - R_p e^{iv_1 h}) \\ & + \varepsilon_{L,z} \mathbf{e}_z (e^{-iv_1 h} + R_p e^{iv_1 h}). \end{aligned} \quad (9)$$

From this representation it follows immediately that without the reflected field this reduces to

$$\mathbf{m}(h) = \mathbf{e}_L e^{-iv_1 h}, \quad (10)$$

which is Eq. (3) at $\mathbf{r}_0 = -H\mathbf{e}_z$, apart from the factor E_0 .

4. REFLECTION OF THE SOURCE FIELD

The source field of the dipole can be represented as an angular spectrum of plane waves. Each partial wave in this representation reflects at the interface, resulting in an angular spectrum representation of the reflected field in the half-space $z < 0$ [26]:

$$\mathbf{E}_r(\mathbf{r}) = \frac{i\mu_1 k_o}{8\pi^2 \varepsilon_o} \sum_{\sigma=s,p} \int d^2 \mathbf{k}_{\parallel} \frac{e^{i h v_1}}{v_1} (\mathbf{d} \cdot \mathbf{e}_{\sigma}) R_{\sigma} \mathbf{e}_{\sigma,r} e^{i \mathbf{k}_r \cdot \mathbf{r}}. \quad (11)$$

We adopt polar coordinates $(k_{\parallel}, \tilde{\phi})$ in the $\mathbf{k}_{\parallel}$ plane, and we change variables as $u = k_{\parallel}/k_o$. The Fresnel reflection coefficients R_s and R_p are functions of u . For each $\mathbf{k}_{\parallel}$, the integrand is a plane wave. The wave vectors are $\mathbf{k}_r = \mathbf{k}_{\parallel} - k_o v_1 \mathbf{e}_z$, with

$$v_1(u) = \sqrt{n_1^2 - u^2}. \quad (12)$$

The branch line for the square root function is taken just below the negative real axis. For $k_{\parallel} < n_1 k_o$ we have $0 \leq u < n_1$, and the wave vector $\mathbf{k}_r$ is real. The wave is a traveling wave, similar to the reflected laser field. Then $u = n_1 \sin \theta_i$, with θ_i the angle of incidence of the corresponding source wave. These Fresnel coefficients are the same as in Eqs. (6) and (8), so there they need to be evaluated at $u = n_1 \sin \theta_i$. For $u > n_1$, the z component of $\mathbf{k}_r$ is imaginary, and the wave is evanescent, decaying exponentially in the direction away from the interface.

The integral over the polar angle $\tilde{\phi}$ is elementary and leads to a host of Bessel functions. The remaining integrals over u are Sommerfeld-type integrals, which can be evaluated numerically [27]. The arguments of the Bessel functions are $u k_o \rho$, with ρ the distance to the z axis. Since we need the field on the z axis, we have $u k_o \rho = 0$, and since $J_n(0) = \delta_{n,0}$, all Bessel functions drop out. For a point on the z axis, we also have $\mathbf{k}_r \cdot \mathbf{r} = -k_o v_1 z$, and at the location of the dipole this is $\mathbf{k}_r \cdot \mathbf{r} = v_1 h$. Therefore, the two exponentials in Eq. (11) combine as $\exp(2i v_1 h)$. We so find for the reflected field at the location of the dipole the simple expression

$$\mathbf{E}_r(\mathbf{r}_o) = \frac{i(n_1 k_o)^3}{8\pi \varepsilon_o \varepsilon_1} [\mathbf{d}_{\parallel} W_{\parallel}(h) + \mathbf{d}_{\perp} W_{\perp}(h)]. \quad (13)$$

Here, $\mathbf{d}_{\parallel}$ and $\mathbf{d}_{\perp}$ are the parallel and perpendicular parts of $\mathbf{d}$ with respect to the surface. The two W functions are

$$W_{\parallel}(h) = \frac{1}{n_1^3} \int_0^{\infty} du \frac{u}{v_1} [n_1^2 R_s(u) - v_1^2 R_p(u)] e^{2i v_1 h}, \quad (14)$$

$$W_{\perp}(h) = \frac{2}{n_1^3} \int_0^{\infty} du \frac{u^3}{v_1} R_p(u) e^{2i v_1 h}. \quad (15)$$

Clearly, these functions of h need to be evaluated numerically.

A complication with the integrals in Eqs. (14) and (15) is the appearance of v_1 in the denominators. We see from Eq. (12) that we have $v_1 = 0$ for $u = n_1$, and this u value is on the integration axis. So, we have a singularity at $u = n_1$. Also, v_1 has a branch point at $u = n_1$, and this appears in the exponential $\exp(2i v_1 h)$, which is numerically not attractive. To resolve this problem, we split the integrals in two parts. The first part is over the range $0 \leq u < n_1$. In this region the waves are traveling. We make the substitution $u = n_1(1 - t^2)^{1/2}$, and this gives

$$W_{\parallel}(h)^{\text{tr}} = \int_0^1 dt e^{i\beta t} [R_s(t) - t^2 R_p(t)], \quad (16)$$

$$W_{\perp}(h)^{\text{tr}} = 2 \int_0^1 dt e^{i\beta t} (1 - t^2) R_p(t), \quad (17)$$

where we have set $\beta = 2n_1 h$. Now the singularities have disappeared, and there is no branch point in the exponentials. Also, the overall numerical factors are gone, and so is the v_1^2 in the integrand of Eq. (14). For the range $n_1 < u < \infty$ the waves are evanescent, and here we make the substitution $u = n_1(1 + t^2)^{1/2}$. This yields the representations

$$W_{\parallel}(h)^{\text{ev}} = -i \int_0^{\infty} dt e^{-\beta t} [R_s(t) + t^2 R_p(t)], \quad (18)$$

$$W_{\perp}(h)^{\text{ev}} = -2i \int_0^{\infty} dt e^{-\beta t} (1 + t^2) R_p(t). \quad (19)$$

In these new representations there are no singularities, which proves that the original $1/v_1$ singularities are integrable. Interesting to note is that Eqs. (18) and (19) are Laplace transforms, with β as the Laplace parameter.

5. FUNCTIONS $W_k(h)$

The reflected electric field of the dipole at the location of the dipole is determined by the functions $W_k(h)$, with $k = \parallel, \perp$ and $W_k(h) = W_k(h)^{\text{tr}} + W_k(h)^{\text{ev}}$. We shall now examine some of the properties of these functions. Without any computations we see that

$$W_k(h) \rightarrow 0, \quad h \rightarrow \infty. \quad (20)$$

The h dependence only comes in through $\beta = 2n_1 h$. For the traveling parts, the exponential $\exp(i\beta t)$ oscillates very rapidly, and these fast oscillations integrate to zero. For the evanescent parts, the exponential $\exp(-\beta t)$ goes to zero. For $h \rightarrow 0$ the traveling integrals are finite, because the integration is over a finite range, but the evanescent integrals diverge in the upper limit. Therefore, for $h \rightarrow 0$ the contributions from the evanescent parts dominate. Let us consider Eq. (16). For small β the main contribution comes from the range of large t values. The Fresnel coefficients level off to a finite value for $t \rightarrow \infty$. The part $t^2 \exp(-\beta t)$ of the integrand has a maximum at $t = 1/\beta$, and for β small, this maximum moves to infinity. So, the term with $R_s(t)$ becomes negligible when compared to the term with $R_p(t)$. For t large, we can set approximately $R_p(t) \approx R_p(\infty)$, and the remaining integral over $t^2 \exp(-\beta t)$ is $2/\beta^3$. Similarly, the term “1” in the integrand of Eq. (17) can be neglected. We introduce δ_k as $\delta_{\parallel} = 1, \delta_{\perp} = 2$. We then find

$$W_k(h) = -\frac{2i\delta_k}{\beta^3} R_p(\infty) + \dots, \quad h \text{ small}, k = \parallel, \perp. \quad (21)$$

Here we dropped the superscript ev , since the contribution from the tr part is on the ellipses. We conclude that both functions diverge for h small, and they diverge as $1/h^3$.

At this point it is useful to have the explicit expressions for the Fresnel coefficients. For a single interface we have

$$R_s(u) = \frac{\mu_2 v_1 - \mu_1 v_2}{\mu_2 v_1 + \mu_1 v_2}, \quad (22)$$

$$R_p(u) = \frac{\varepsilon_2 v_1 - \varepsilon_1 v_2}{\varepsilon_2 v_1 + \varepsilon_1 v_2}, \quad (23)$$

and here we only need R_p . For the reflection of the laser beam we have $u = n_1 \sin \theta_i$, but here we also need the extension to the evanescent region. We have

$$n_2^2 = \varepsilon_2 \mu_2, \quad (24)$$

$$v_2(u)^2 = n_2^2 - u^2, \quad (25)$$

for the index of refraction in medium 2, and for the dimensionless z component of the wave vector in medium 2. Care should be exercised in taking the square roots when ε_2 and μ_2 are complex. We need the values of n_2 and v_2 for which the imaginary parts are non-negative. It can be shown that the correct roots are

$$n_2 = \sqrt{\varepsilon_2} \sqrt{\mu_2}, \quad (26)$$

$$v_2(u) = \sqrt{n_2 + u} \sqrt{n_2 - u}. \quad (27)$$

For n_1 and v_1 there is no issue since we assume that both ε_1 and μ_1 are positive. Equations (22) and (23) express the Fresnel coefficients as a function of u . To evaluate the integrals in Eqs. (16)–(19), the appropriate substitutions $u \rightarrow t$ need to be made.

To find $R_p(\infty)$ from Eq. (23), we notice that we are far in the evanescent region. We then have $v_2^2 = n_2^2 - n_1^2(1 + t^2)$, and for t large this gives $v_2 \approx i n_1 t \approx v_1$. We then obtain

$$R_p(\infty) = \frac{\varepsilon_2 - \varepsilon_1}{\varepsilon_2 + \varepsilon_1}. \quad (28)$$

When considering a layer, as in Fig. 1, a similar calculation gives the same result as in Eq. (28). This can be understood from the fact that evanescent waves emanating from the interface at $z = 0$ do not reach the second interface. The real and imaginary parts are

$$\text{Re } R_p(\infty) = \frac{1}{|\varepsilon_2 + \varepsilon_1|^2} (|\varepsilon_2|^2 - \varepsilon_1^2), \quad (29)$$

$$\text{Im } R_p(\infty) = \frac{2\varepsilon_1}{|\varepsilon_2 + \varepsilon_1|^2} \text{Im } \varepsilon_2. \quad (30)$$

Since $\text{Im } \varepsilon_2 \geq 0$, we have $\text{Im } R_p(\infty) \geq 0$. For $\text{Im } \varepsilon_2 > 0$ we then see that $\text{Re } W_k(h) \rightarrow +\infty$ for $h \rightarrow 0$. The imaginary part diverges to $-\infty$ for $|\varepsilon_2| > \varepsilon_1$ and to $+\infty$ for $|\varepsilon_2| < \varepsilon_1$. A typical example of a W function is shown in Fig. 2. The function diverges for h small and oscillates for larger h values, eventually going to zero. In all of the following graphs we shall consider a single interface and set $\mu_1 = \mu_2 = 1$.

An interesting special case is a perfectly conducting material (mirror). Then we have $R_s = -1$, $R_p = 1$, and the integrals in Eqs. (16)–(19) can be found in closed form. We then obtain

$$W_{\parallel}(h) = \frac{2i}{\beta} \left(1 + \frac{i}{\beta} - \frac{1}{\beta^2} \right) e^{i\beta}, \quad (31)$$

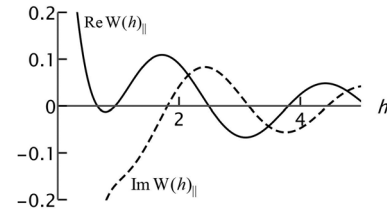


Fig. 2. Real (solid curve) and imaginary (dashed curve) parts of the function $W_{\parallel}(h)$ for a single interface with $\varepsilon_1 = 1.5$ and $\varepsilon_2 = 4.5$.

$$W_{\perp}(h) = -\frac{4}{\beta^2} \left(1 + \frac{i}{\beta} \right) e^{i\beta}. \quad (32)$$

The oscillatory behavior comes from the factors $\exp(i\beta)$, and the limit $h \rightarrow 0$ agrees with Eq. (21) with $R_p = 1$.

6. DIPOLE MOMENT

The induced dipole moment $\mathbf{d}$ is determined by the external field at the location of the particle, as in Eq. (2). The external field is the sum of the laser field and its reflection and the reflected dipole radiation. So we have

$$\mathbf{d} = \alpha [\mathbf{E}_{L+R}(\mathbf{r}_0) + \mathbf{E}_r(\mathbf{r}_0)]. \quad (33)$$

A convenient quantity is the dimensionless polarizability volume, defined as

$$\bar{V}_p = \frac{(n_1 k_0)^3}{4\pi \varepsilon_0 \varepsilon_1} \alpha. \quad (34)$$

With Eqs. (7) and (13) we then have

$$\mathbf{d} = \alpha E_o \mathbf{m}(h) + \frac{i}{2} \bar{V}_p [\mathbf{d}_{\parallel} W_{\parallel}(h) + \mathbf{d}_{\perp} W_{\perp}(h)]. \quad (35)$$

Since $\mathbf{d}$ also appears on the right-hand side, this is an equation for $\mathbf{d}$. We split this vector equation in its parallel and perpendicular parts, which gives

$$\mathbf{d}_k = \alpha E_o \mathbf{m}(h)_k + \frac{i}{2} \bar{V}_p \mathbf{d}_k W_k(h), \quad k = \parallel, \perp. \quad (36)$$

The solution of this equation is

$$\mathbf{d}_k = \Upsilon_k(h) \mathbf{m}(h)_k \alpha E_o, \quad k = \parallel, \perp, \quad (37)$$

where we have introduced the two resolvent functions

$$\Upsilon_k(h) = \frac{1}{1 - \frac{i}{2} \bar{V}_p W_k(h)}, \quad k = \parallel, \perp. \quad (38)$$

The final expression for the dipole moment then becomes

$$\mathbf{d} = [\Upsilon_{\parallel}(h) \mathbf{m}(h)_{\parallel} + \Upsilon_{\perp}(h) \mathbf{m}(h)_{\perp}] \alpha E_o. \quad (39)$$

The result (39) has a transparent interpretation. The functions $\Upsilon_{\parallel}(h)$ and $\Upsilon_{\perp}(h)$ account for the contribution of the reflected dipole radiation to $\mathbf{d}$. Without this contribution we would have $\Upsilon_{\parallel}(h) = 1$ and $\Upsilon_{\perp}(h) = 1$. The functions $\mathbf{m}(h)_{\parallel}$ and $\mathbf{m}(h)_{\perp}$ account for the reflected laser light. Without this reflection, the function $\mathbf{m}(h)$ is given by Eq. (10). So, without the surface we would have

$$\mathbf{d} = \alpha E_o \mathbf{e}_L e^{-i v_1 h}. \quad (40)$$

The functions $\mathbf{m}(h)$ and $\Upsilon_k(h)$ then modify this expression to include the laser field and the dipole field reflection at the interface, respectively. The result for $\Upsilon_{\perp}(h)$ was derived in Ref. [9] for a perpendicular dipole moment in vacuum and very close to the interface with a perfect conductor. We then have $W_{\perp}(h) \approx -4i/\beta^3$ with Eq. (32), and our result agrees with Ref. [9] in this limit. They consider the dipole moment as a function of the frequency ω for a given h , whereas we consider the h dependence for a given ω .

7. POLARIZABILITY

The properties of the particle enter the solution through the polarizability α , or equivalently, the (dimensionless) polarizability volume $\bar{\mathcal{V}}_p$. Before proceeding with the specification of α , we would like to point out that we cannot just adopt any model for α . Let us consider a particle in a laser beam and embedded in an ε_1, μ_1 medium. The dipole moment is then given by Eq. (40). The power emitted by the source can be computed by integrating the Poynting vector over a sphere with the particle in the center. With the known expressions for the electric and magnetic fields of a dipole, this gives [28]

$$P_s = \frac{1}{2} \omega \frac{(n_1 k_o)^3}{6\pi \varepsilon_o \varepsilon_1} d_o^2, \quad (41)$$

and here we have $d_o^2 = \mathbf{d}^* \cdot \mathbf{d} = |\alpha|^2 E_o^2$ with Eq. (40). However, the power absorbed from the laser field is on general grounds ([25], p. 266):

$$P_a = -\frac{1}{2} \omega \text{Im}[\mathbf{d}^* \cdot \mathbf{E}_L(\mathbf{r}_o)]. \quad (42)$$

With Eqs. (40) and (3) this becomes

$$P_a = \frac{1}{2} \omega E_o^2 \text{Im}\alpha. \quad (43)$$

If no energy accumulates in the particle, then clearly we must have $P_a = P_s$. This is only possible if

$$\text{Im}\alpha = \frac{(n_1 k_o)^3}{6\pi \varepsilon_o \varepsilon_1} |\alpha|^2. \quad (44)$$

This equation only involves the particle, and it relates the imaginary part of α to its absolute value. Apparently, this relation must hold when no energy is dissipated by the particle. With Eq. (34) this can be expressed in a relation for the polarization volume:

$$\text{Im}\bar{\mathcal{V}}_p = \frac{2}{3} |\bar{\mathcal{V}}_p|^2. \quad (45)$$

Since the right-hand side is non-negative, we have

$$\text{Im}\bar{\mathcal{V}}_p \geq 0, \quad (46)$$

and since $\text{Im}\bar{\mathcal{V}}_p \leq |\bar{\mathcal{V}}_p|$, we find

$$|\bar{\mathcal{V}}_p| \leq \frac{3}{2}. \quad (47)$$

So, on very general grounds, the absolute value of $\bar{\mathcal{V}}_p$ has an upper bound. With Eq. (34) this implies the upper bound for $|\alpha|$:

$$|\alpha| \leq \frac{6\pi \varepsilon_o \varepsilon_1}{(n_1 k_o)^3}. \quad (48)$$

For a particle in vacuum and a laser wavelength of 500 nm, this gives $|\alpha| \leq 8.34 \times 10^{-32} \text{ C} \cdot \text{m}^2/\text{V}$, and this upper bound is independent of any properties of the particle. Relations (45)–(47) for $\bar{\mathcal{V}}_p$ do not involve any constants, which shows the advantage of working with dimensionless variables and functions.

Relation (44) was derived by considering an embedded dipole in a laser beam. It is shown in Appendix A that this relation also guarantees conservation of energy for the problem involving the surface, as considered here.

Let us now consider a dielectric sphere with radius R and permittivity ε_p , assumed to be real. It follows from exact Mie theory that in the limit of a small particle ($n_1 k_o R \ll 1$; the long-wavelength approximation, or zero-frequency limit) that the polarizability is [29]

$$\alpha_o = 4\pi \varepsilon_o \varepsilon_1 \eta R^3, \quad (49)$$

with corresponding polarization volume

$$\bar{\mathcal{V}}_{p,o} = \eta (n_1 \bar{R})^3. \quad (50)$$

Here, $\bar{R} = k_o R$ is the dimensionless radius of the particle. The parameter η is defined as

$$\eta = \frac{\varepsilon_p - \varepsilon_1}{\varepsilon_p + 2\varepsilon_1}. \quad (51)$$

It is obvious that α_o does not satisfy Eq. (44), necessary for energy conservation. It was derived by Draine [30] that for $\omega \neq 0$ the expression for α should be replaced by

$$\alpha = \frac{\alpha_o}{1 - \frac{2}{3} i \eta (n_1 k_o R)^3}. \quad (52)$$

It readily verifies that this α satisfies Eq. (44). This expression was derived to salvage the validity of the optical theorem for scattering cross sections. As shown earlier, it also guarantees energy conservation for non-absorbing particles (ε_p real). A more elaborate expression was introduced in [23,24] to include effects of the finite radius R . In terms of the polarizability volume, Eq. (52) reads

$$\bar{\mathcal{V}}_p = \frac{\bar{\mathcal{V}}_{p,o}}{1 - \frac{2}{3} i \bar{\mathcal{V}}_{p,o}}, \quad (53)$$

with $\bar{\mathcal{V}}_{p,o}$ given by Eq. (50). This expression is to be used in the resolvents $\Upsilon_k(h)$, Eq. (38). The case of a conducting particle follows by setting $\eta = 1$. The parameter η , defined in Eq. (51), diverges for $\varepsilon_p \rightarrow -2\varepsilon_1$, which may seem cumbersome. However, this is just an artifact of the representation in terms of η . It follows immediately from Eqs. (50) and (53) that

$$\bar{\mathcal{V}}_p \rightarrow \frac{3}{2} i, \quad \varepsilon_p \rightarrow -2\varepsilon_1. \quad (54)$$

8. FUNCTIONS $\Upsilon_k(h)$

The resolvents $\Upsilon_k(h)$, given by Eq. (38), account for the effect of the reflected dipole radiation. Far away from the interface

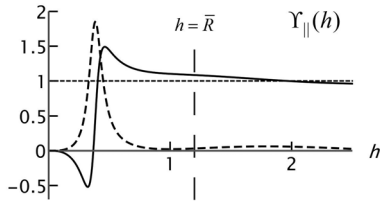


Fig. 3. Real (solid curve) and imaginary (dashed curve) parts of the function $\Upsilon_{\parallel}(h)$ for a dielectric particle of radius $\bar{R} = 1.2$ and for $\varepsilon_p = 3$, $\varepsilon_1 = 1$, and $\varepsilon_2 = 6$. Deviations from the dotted line at unity represent effects due to the reflected dipole radiation. Without this effect we would have $\text{Re}\Upsilon_{\parallel}(h) = 1$ and $\text{Im}\Upsilon_{\parallel}(h) = 0$.

we have $W_k(h) \rightarrow 0$, and so $\Upsilon_k(h) \rightarrow 1$ for $h \rightarrow \infty$, as could be expected. Any deviation of $\Upsilon_k(h)$ from unity is due to the reflected dipole radiation. For intermediate h , the functions $W_k(h)$ are oscillatory, and so the functions $\Upsilon_k(h)$ will be oscillatory. The main effect is expected to appear when the dipole is close to the interface. The values of $W_k(h)$ for h small are given by Eq. (21), and they diverge. The corresponding values for $\Upsilon_k(h)$ then become

$$\Upsilon_k(h) = \frac{\varepsilon_1 + \varepsilon_2}{\varepsilon_1 - \varepsilon_2} \frac{1}{\delta_k \bar{V}_p} \beta^3 [1 + O(\beta)], \quad h \text{ small.} \quad (55)$$

We notice that $\Upsilon_k(h) \rightarrow 0$ as h^3 , no matter the type of the particle.

A typical example of an Υ function for a dielectric particle is shown in Fig. 3. The particle cannot come closer to the interface than its own radius, so the curves only have significance for $h > \bar{R}$. The borderline $h = \bar{R}$ is indicated by the vertical dashed line. We see from the figure that $\Upsilon_{\parallel}(h)$ has a substantial structure in the region $h < \bar{R}$, but this region is not accessible to the particle. For $h > \bar{R}$ we have $\Upsilon_{\parallel}(h) \approx 1$, so the reflected dipole radiation has as good as no effect here.

It follows from Eq. (38) that we get a division by zero, or a resonance, if

$$\bar{V}_p W_k(h_o) = -2i, \quad (56)$$

for a certain h_o . From Eq. (47) we have that $\bar{V}_p$ is bounded by $|\bar{V}_p| \leq 3/2$, and $W_k(h)$ goes to zero for h large. Therefore, this equation can only have solutions for relatively small h_o . For estimation purposes, we use Eq. (21). We then obtain

$$\delta_k R_p(\infty) \bar{V}_p \approx \beta_o^3, \quad (57)$$

with solution

$$h_o \approx \frac{1}{2n_1} \sqrt[3]{\delta_k R_p(\infty) \bar{V}_p}, \quad (58)$$

if such a solution exists. If $\Upsilon_{\parallel}(h)$ has a peak for a certain h_o , then $\Upsilon_{\perp}(h)$ also has a peak, but at an h value of $\sqrt[3]{2} = 1.26$ times larger. So, a $\perp$ peak is always to the right of a $\parallel$ peak. The peak heights are unrelated because these are determined by the ellipses in Eq. (21).

For a dielectric sphere the polarizability volume is given by Eq. (53). We are looking for solutions with h small. This implies $\bar{R}$ small, otherwise the sphere cannot come close enough to the surface. We then have $\bar{V}_p \approx \bar{V}_{p,o} = \eta(n_1 \bar{R})^3$. Equation (58) becomes for $k = \parallel$:

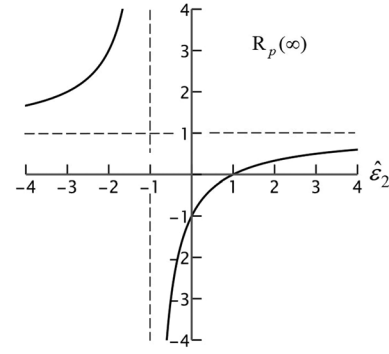


Fig. 4. The figure shows $R_p(\infty)$ as a function of $\hat{\varepsilon}_2 = \varepsilon_2/\varepsilon_1$.

$$h_o \approx \frac{1}{2} \bar{R} \sqrt[3]{\eta R_p(\infty)}. \quad (59)$$

Here, η is given by Eq. (51) and $R_p(\infty)$ by Eq. (28). Assuming ε_2 to be real, so that $R_p(\infty)$ is real, we see that we can only have a solution if η and $R_p(\infty)$ have the same sign. Furthermore, we must have $h_o > \bar{R}$, which gives the restriction

$$\eta R_p(\infty) > 8, \quad (60)$$

for a $\parallel$ resonance to appear. For instance, for a conducting substrate we have $R_p(\infty) = 1$ and for a conducting particle we have $\eta = 1$. Clearly, there is no solution for $h_o > \bar{R}$. We shall now look at combinations of parameters for which a solution exists.

The search for resonances is facilitated by Figs. 4 and 5, where $R_p(\infty)$ and η are graphed as functions of $\hat{\varepsilon}_2 = \varepsilon_2/\varepsilon_1$ and $\hat{\varepsilon}_p = \varepsilon_p/\varepsilon_1$, respectively. We can only have peaks in $\Upsilon_k(h)$ if $R_p(\infty)$ and η have the same sign. This leaves five possibilities. The most obvious case is $\hat{\varepsilon}_2 > 1$, $\hat{\varepsilon}_p > 1$, corresponding to a dielectric medium and particle. Then $0 < R_p(\infty) < 1$, $0 < \eta < 1$, so $\eta R_p(\infty) < 1$. We then see with Eq. (59) that for this case there are no solutions for $h > \bar{R}$. The second possibility is $\hat{\varepsilon}_2 > 1$, $\hat{\varepsilon}_p < -2$. Then $0 < R_p(\infty) < 1$, $\eta > 1$, and this could be possible. It should be noted that “possible” means that for certain values in the given range there may be a solution, but it does not imply that for all parameters in this range there is a solution. For instance, for $\varepsilon_p \rightarrow -\infty$ ($\eta = 1$; conducting particle) there is no solution satisfying Eq. (59). Figure 6 shows $|\Upsilon_{\parallel}(h)|^2$ and $|\Upsilon_{\perp}(h)|^2$ for $\varepsilon_1 = 1$, $\varepsilon_2 = 30$, $\varepsilon_p = -2.15$, and $\bar{R} = 0.25$. For these values we have $R_p(\infty) = 0.935$, $\eta = 21$. Then $R_p(\infty)\eta = 19.6$, and with Eq. (59) we find that the $\parallel$ resonance should be located at $h_o \approx 0.34$, and the $\perp$ resonance should then be located at $1.26 \times 0.34 = 0.43$. This is in good agreement with the graphs. The curves should be compared to the dotted line at unity in Fig. 6, which represents the value if the reflected dipole radiation would not have been taken into account. We notice that the peaks are huge, and in particular the $\parallel$ resonance gives an enhancement of about 15 for these parameters. Any damping in ε_2 or ε_p (positive imaginary parts) will in general lead to a reduction of peak heights.

The third possibility is $\hat{\varepsilon}_2 < -1$, $\hat{\varepsilon}_p > 1$. Other possibilities are $\hat{\varepsilon}_2 < -1$, $\hat{\varepsilon}_p < -2$, and $-1 < \hat{\varepsilon}_2 < 1$, $-2 < \hat{\varepsilon}_p < 1$. The graphs are all very similar to Fig. 6. Usually, the $\parallel$ resonance is the strongest, but there are exceptions. There is also the possibility that the $\parallel$ peak is to the left of $h = \bar{R}$ and the $\perp$ peak is to

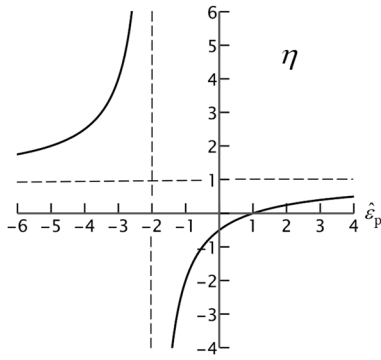


Fig. 5. The graph shows η as a function of $\hat{\epsilon}_p = \epsilon_p/\epsilon_1$.

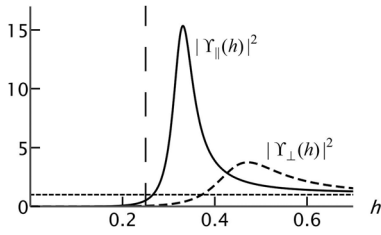


Fig. 6. Shown are $|\Upsilon_{||}(h)|^2$ (solid line) and $|\Upsilon_{\perp}(h)|^2$ (dashed line) as a function of h and for the parameters given in the text. The particle can only be located to the right of the vertical dashed line.

the right. In this case there is effectively only one resonance. It should finally be noted that the presence of these resonances is the exception rather than the rule. They only appear for specific values of the parameters.

9. FUNCTION $d_o(h)^2$

The solution for the dipole moment, given by Eq. (39), is a complex-valued vector $\mathbf{d}$. In the results for the power (next section), this vector only comes in as

$$d_o(h)^2 = \mathbf{d}^* \cdot \mathbf{d}. \quad (61)$$

We have

$$d_o(h)^2 =$$

$$[|\Upsilon_{||}(h)|^2 \mathbf{m}(h)_{||}^* \cdot \mathbf{m}(h)_{||} + |\Upsilon_{\perp}(h)|^2 \mathbf{m}(h)_{\perp}^* \cdot \mathbf{m}(h)_{\perp}] |\alpha|^2 E_o^2, \quad (62)$$

and there are no cross terms between $||$ and $\perp$. Without the surface, this would be $d_o(h)^2 = |\alpha|^2 E_o^2$. The functions $\mathbf{m}_k(h)$ account for the laser reflection, and this gives undamped oscillations as a function of h . The functions $\Upsilon_k(h)$ include the effect of the reflected dipole radiation. We graph $d_o(h)^2/(|\alpha|^2 E_o^2)$, so that any deviation from unity is due to the surface. The solid curves are the solutions given by Eq. (62), and the dashed curves are the solutions with $\Upsilon_k(h) = 1$. So, any difference between solid curves and dashed curves is due to the reflected dipole radiation. For h small, we have $d_o(h)^2 \sim h^6$, since $\Upsilon_k(h) \sim h^3$.

Figure 7 shows a typical example. The radius is $\bar{R} = 1.5$, $\epsilon_p = 3$, $\epsilon_1 = 1$, and $\epsilon_2 = 5$. The angle of incidence is $\theta_i = 60^\circ$, and we have s polarization ($a_s = 1$, $a_p = 0$). We see that there is hardly any difference between the solid and dashed curves in

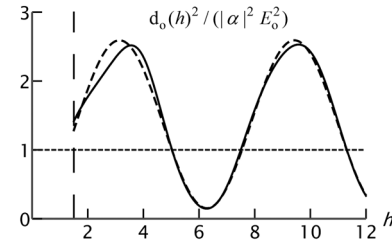


Fig. 7. The figure shows $d_o(h)^2/(|\alpha|^2 E_o^2)$ with (solid line) and without (dashed line) the function $\Upsilon_k(h)$, as a function of h . The parameters are given in the text.

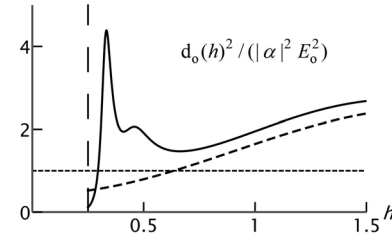


Fig. 8. The figure shows $d_o(h)^2/(|\alpha|^2 E_o^2)$ for the parameters given in the text.

the region $h > \bar{R}$, so the reflected dipole radiation has as good as no effect here. A more interesting result is shown in Fig. 8. The parameters are $\bar{R} = 0.25$, $\epsilon_p = -2.15$, $\epsilon_1 = 1$, $\epsilon_2 = 30$, $\theta_i = 60^\circ$, and $a_s = a_p = 1$ (circular polarization). For h slightly larger than $\bar{R}$, a peak appears, which is due to the resonances in $\Upsilon_k(h)$. The larger peak is the $||$ resonance, and the smaller peak on the right is due to $\Upsilon_{\perp}(h)$.

10. POWER EMISSION

The emitted power is determined by the oscillating dipole moment and its environment. It has two contributions:

$$P_e(h) = P_s(h) + P_r(h). \quad (63)$$

For a dielectric particle it is more common to call this the scattered power. The $P_s(h)$ is the emitted power due to the source field and is given by Eq. (41). For an embedded dipole, but without the interface, we have $d_o^2 = \mathbf{d}^* \cdot \mathbf{d} = |\alpha|^2 E_o^2$, as in Section 7, but with the interface this becomes d_o^2 from Eq. (62). This gives for the source power

$$P_s(h) = \mu_1 n_1 P_f [|\Upsilon_{||}(h)|^2 \mathbf{m}(h)_{||}^* \cdot \mathbf{m}(h)_{||} + |\Upsilon_{\perp}(h)|^2 \mathbf{m}(h)_{\perp}^* \cdot \mathbf{m}(h)_{\perp}]. \quad (64)$$

Here we have introduced

$$P_f = \frac{\omega k_o^3}{12\pi \epsilon_o} |\alpha|^2 |E_o|^2, \quad (65)$$

as the emitted source power for a dipole in a laser beam, but in free space. Then $\mu_1 n_1 P_f$ is the emitted power for an embedded dipole in a laser beam.

The term $P_r(h)$ in Eq. (63) is the modification of the power emission due to the reflected dipole field, and is given by

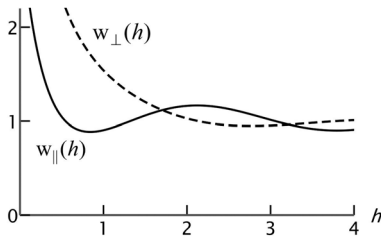


Fig. 9. Functions $w_{\parallel}(h)$ and $w_{\perp}(h)$ for $\varepsilon_1 = 1$ and $\varepsilon_2 = 10 + 0.001 \cdot i$.

$$P_r(h) = \frac{1}{2} \omega \text{Im}[\mathbf{d}^* \cdot \mathbf{E}_r(\mathbf{r}_0)]. \quad (66)$$

With $\mathbf{d}$ given by Eq. (39) and $\mathbf{E}_r(\mathbf{r}_0)$ by Eq. (13), we find

$$P_r(h) = \frac{3}{4} \mu_1 n_1 P_f \{ |\Upsilon_{\parallel}(h)|^2 [\mathbf{m}(h)_{\parallel}^* \cdot \mathbf{m}(h)_{\parallel}] \text{Re} W_{\parallel}(h) + |\Upsilon_{\perp}(h)|^2 [\mathbf{m}(h)_{\perp}^* \cdot \mathbf{m}(h)_{\perp}] \text{Re} W_{\perp}(h) \}. \quad (67)$$

Adding Eqs. (64) and (67) yields the final result:

$$P_e(h) = \mu_1 n_1 P_f \{ |\Upsilon_{\parallel}(h)|^2 [\mathbf{m}(h)_{\parallel}^* \cdot \mathbf{m}(h)_{\parallel}] w_{\parallel}(h) + |\Upsilon_{\perp}(h)|^2 [\mathbf{m}(h)_{\perp}^* \cdot \mathbf{m}(h)_{\perp}] w_{\perp}(h) \}. \quad (68)$$

The functions $w_k(h)$ are defined as

$$w_k(h) = 1 + \frac{3}{4} \text{Re} W_k(h), \quad k = \parallel, \perp. \quad (69)$$

Comparison with Eq. (64) for the power emitted by the source field shows that the contributions from the reflected dipole radiation (interference) are accounted for by the functions $w_{\parallel}(h)$ and $w_{\perp}(h)$. These functions are identical to the radiative decay rates for a molecule near an interface with a parallel or perpendicular dipole moment [5].

For h large we have $w_k(h) \rightarrow 1$, since $W_k(h) \rightarrow 0$, and for intermediate values of h the functions $w_k(h)$ are oscillatory. The behavior for small h follows from Eqs. (21) and (30):

$$w_k(h) = \frac{3\delta_k}{\beta^3} \frac{\varepsilon_1}{|\varepsilon_2 + \varepsilon_1|^2} \text{Im} \varepsilon_2 + \dots, \quad h \rightarrow 0. \quad (70)$$

The functions diverge to $+\infty$ as $1/h^3$ for $h \rightarrow 0$ if ε_2 has an imaginary part, and $w_{\perp}(h) \approx 2w_{\parallel}(h)$. If $\text{Im} \varepsilon_2 = 0$, the approach to $h \rightarrow 0$ is undetermined by Eq. (70). Figure 9 illustrates $w_{\parallel}(h)$ and $w_{\perp}(h)$ for $\text{Im} \varepsilon_2 \neq 0$. We have $\varepsilon_2 = 10 + 0.001 \cdot i$, and we see that even this small $\text{Im} \varepsilon_2$ makes the functions diverge for $h \rightarrow 0$. In Fig. 10 we have $\varepsilon_2 = -1.3$, so $\text{Im} \varepsilon_2 = 0$, and we see that both $w_{\parallel}(h)$ and $w_{\perp}(h)$ remain finite for $h \rightarrow 0$.

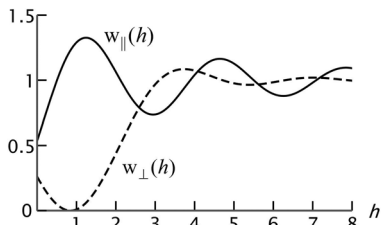


Fig. 10. Functions $w_{\parallel}(h)$ and $w_{\perp}(h)$ for $\varepsilon_1 = 1$ and $\varepsilon_2 = -1.3$.

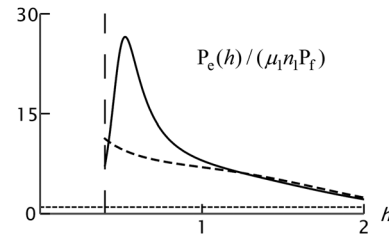


Fig. 11. Normalized power for a dielectric sphere with $\bar{R} = 0.4$. The other parameters are given in the text.

Equations (31) and (32) give the functions $W_{\parallel}(h)$ and $W_{\perp}(h)$ for a perfect conductor. With Eq. (69) we then find for the $w_k(h)$ functions

$$w_{\parallel}(h) = 1 - \frac{3}{2\beta} \left[\frac{1}{\beta} \cos \beta + \left(1 - \frac{1}{\beta^2} \right) \sin \beta \right], \quad (71)$$

$$w_{\perp}(h) = 1 - \frac{3}{\beta^2} \left(\cos \beta - \frac{1}{\beta} \sin \beta \right), \quad (72)$$

and expanding for β small gives

$$w_{\parallel}(h) = \frac{1}{5} \beta^2 + \dots, \quad (73)$$

$$w_{\perp}(h) = 2 - \frac{1}{10} \beta^2 + \dots \quad (74)$$

Rather than diverging for $h \rightarrow 0$, as in Eq. (70), these functions remain finite for a perfect conductor.

The h dependence of $P_e(h)$ comes in through the functions $\mathbf{m}(h)$, $\Upsilon_k(h)$, and $w_k(h)$, representing the laser and its reflection, the surface-modified dipole moment, and the influence of interference, respectively. Most interesting is the behavior of $P_e(h)$ for h small. When the dipole moment $\mathbf{d}$ is assumed to be given, the h dependence only enters through $w_k(h)$. These functions diverge as $1/h^3$ when there is damping in the material, whereas for $\text{Im} \varepsilon_2 = 0$ these functions may remain finite. When the laser excitation mechanism is taken into consideration, the function $\mathbf{m}(h)$ contributes. This function is finite at all distances, so it does not alter the behavior of the power for $h \rightarrow 0$. When the effect of the reflected dipole radiation on the value of $\mathbf{d}$ is taken into account, we get factors of $|\Upsilon_k(h)|^2$, and with Eq. (55) these factors go to zero as h^6 . Combined with the h dependence of $w_k(h)$, this gives $P_e(h) \approx h^3$ or higher order. For a perfect conductor we have $P_e(h) \approx h^6$. So, instead of diverging for $h \rightarrow 0$, the power goes to zero quickly when the surface contribution to the induced dipole moment is taken into consideration.

We normalize the power as $P_e(h)/(\mu_1 n_1 P_f)$, so that any deviation from unity is due to the presence of the interface. Figure 11 shows the emitted power as a function of the distance h to the interface. The parameters are $\varepsilon_1 = 1$, $\varepsilon_2 = -2 + 0.1 \cdot i$, $\bar{R} = 0.4$, $\varepsilon_p = -3$, $\theta_i = 30^\circ$, $a_s = 1$, and $a_p = 1$. We have $R_p(\infty) = 3$ and $\eta = 4$. With Eq. (59) we expect a peak due to $\Upsilon_{\parallel}(h)$ at $h_0 \approx 0.46$, which agrees reasonably well with the graph. A possible peak due to $\Upsilon_{\perp}(h)$ is absent. It can be shown that this is due to the fact that $\Upsilon_{\perp}(h)$ does not have a pronounced maximum for these parameters.

The dashed curve is the power with $|\Upsilon_k(h)|^2$ set equal to unity, so the deviation of the solid curve from the dashed curve is due to the contribution of the reflected dipole radiation to the dipole moment.

11. CONCLUSIONS

The electric dipole moment of a small particle near an interface is induced by a laser and its reflection at the interface and by the reflected dipole radiation. In most theoretical approaches, including our own, the dipole moment $\mathbf{d}$ is assumed to be given. When the laser and its reflection provide the mechanism for inducing the dipole moment, such an approach is justified, since this decouples from the emission of radiation by the dipole. However, when the particle is close to the interface, the reflected dipole radiation also acts on the particle as an external field, and it adds to the induced dipole moment. We have derived an expression for the induced dipole moment, Eq. (39), which takes into account the reflected dipole radiation. This does not simply add to the dipole moment, since the reflection modifies its own source. We have shown that this effect can be included by means of resolvent functions $\Upsilon_{\parallel}(h)$ and $\Upsilon_{\perp}(h)$, Eq. (38), which depend on the dimensionless distance h between the particle and the interface.

Due to the back action of the dipole radiation on its own source, the resolvent functions have a resonant structure. It appears that two, one, or no resonances can be present when the particle is close enough to the surface. Whether such resonances occur depends on the parameters of the problem. These resonances are typically in the region where h is smaller than the radius of the particle. Since a sphere cannot come closer to the surface than its own radius, these resonances have in general no effect, but exceptions are possible (Figs. 6 and 8).

We have derived an expression for the power emitted by the dipole, which includes the effect of the reflected dipole radiation on the induction of the dipole moment. The resonances in the functions $\Upsilon_{\parallel}(h)$ and $\Upsilon_{\perp}(h)$ may appear in the h dependence of the power, although this is modified by the polarization of the laser. In particular, for s polarization only $\Upsilon_{\parallel}(h)$ contributes, so we have at most one peak due to the resonances.

APPENDIX A

The power emitted by the dipole is given by Eq. (68). One would expect that this power is supplied by the laser beam. We shall now verify this. On general grounds, the absorbed power is

$$P_a = -\frac{1}{2}\omega \text{Im}[\mathbf{d}^* \cdot \mathbf{E}_{L+R}(\mathbf{r}_o)]. \quad (\text{A1})$$

With Eq. (7) this becomes

$$P_a = -\frac{1}{2}\omega E_o \text{Im}[\mathbf{d}^* \cdot \mathbf{m}(h)]. \quad (\text{A2})$$

With $\mathbf{d}$ from Eq. (39) we have

$$\begin{aligned} \mathbf{d}^* \cdot \mathbf{m}(h) &= \{\Upsilon_{\parallel}(h)^* [\mathbf{m}(h)_{\parallel}^* \cdot \mathbf{m}(h)_{\parallel}] \\ &+ \Upsilon_{\perp}(h)^* [\mathbf{m}(h)_{\perp}^* \cdot \mathbf{m}(h)_{\perp}]\} \alpha^* E_o. \end{aligned} \quad (\text{A3})$$

Then we substitute this in Eq. (A2), which gives

$$\begin{aligned} P_a &= \frac{1}{2}\omega E_o^2 \{ [\mathbf{m}(h)_{\parallel}^* \cdot \mathbf{m}(h)_{\parallel}] \text{Im}[\alpha \Upsilon_{\parallel}(h)] \\ &+ [\mathbf{m}(h)_{\perp}^* \cdot \mathbf{m}(h)_{\perp}] \text{Im}[\alpha \Upsilon_{\perp}(h)] \}. \end{aligned} \quad (\text{A4})$$

Now we need to work out the factors $\text{Im}[\alpha \Upsilon_k(h)]$. It follows from the definition (38) that

$$\Upsilon_k(h) = |\Upsilon_k(h)|^2 \times \left[1 + \frac{i}{2} \bar{\mathcal{V}}_p^* W_k(h)^* \right]. \quad (\text{A5})$$

With Eq. (34) we can express α in terms of $\bar{\mathcal{V}}_p$:

$$\alpha = \frac{4\pi \varepsilon_o \varepsilon_1}{(n_1 k_o)^3} \bar{\mathcal{V}}_p, \quad (\text{A6})$$

and with Eq. (A5) this gives

$$\alpha \Upsilon_k(h) = |\Upsilon_k(h)|^2 \frac{4\pi \varepsilon_o \varepsilon_1}{(n_1 k_o)^3} \times \left[\bar{\mathcal{V}}_p + \frac{i}{2} |\bar{\mathcal{V}}_p|^2 W_k(h)^* \right]. \quad (\text{A7})$$

Taking the imaginary part yields

$$\begin{aligned} \text{Im}[\alpha \Upsilon_k(h)] &= |\Upsilon_k(h)|^2 \frac{4\pi \varepsilon_o \varepsilon_1}{(n_1 k_o)^3} \\ &\times \left\{ \text{Im}(\bar{\mathcal{V}}_p) + \frac{1}{2} |\bar{\mathcal{V}}_p|^2 \text{Re}[W_k(h)] \right\}. \end{aligned} \quad (\text{A8})$$

Equation (45) was derived under the assumption that no energy accumulates in the particle for the case of an embedded dipole in a laser beam, but without an interface. Since this relation only involves $\bar{\mathcal{V}}_p$, it should also hold here. Eliminating $\text{Im}(\bar{\mathcal{V}}_p)$ from Eq. (A8) gives

$$\begin{aligned} \text{Im}[\alpha \Upsilon_k(h)] &= |\Upsilon_k(h)|^2 \frac{4\pi \varepsilon_o \varepsilon_1}{(n_1 k_o)^3} \\ &\times |\bar{\mathcal{V}}_p|^2 \frac{2}{3} \left\{ 1 + \frac{3}{4} \text{Re}[W_k(h)] \right\}, \end{aligned} \quad (\text{A9})$$

and with Eq. (64) this becomes

$$\text{Im}[\alpha \Upsilon_k(h)] = |\Upsilon_k(h)|^2 \frac{4\pi \varepsilon_o \varepsilon_1}{(n_1 k_o)^3} \times |\bar{\mathcal{V}}_p|^2 \frac{2}{3} w_k(h). \quad (\text{A10})$$

Then we eliminate $|\bar{\mathcal{V}}_p|^2$ in favor of $|\alpha|^2$ with Eq. (34):

$$\text{Im}[\alpha \Upsilon_k(h)] = \frac{(n_1 k_o)^3}{6\pi \varepsilon_o \varepsilon_1} \times |\alpha|^2 |\Upsilon_k(h)|^2 w_k(h). \quad (\text{A11})$$

Substitution into Eq. (A4) yields

$$\begin{aligned} P_a &= \omega \frac{(n_1 k_o)^3}{12\pi \varepsilon_o \varepsilon_1} |\alpha|^2 E_o^2 \{ [\mathbf{m}(h)_{\parallel}^* \cdot \mathbf{m}(h)_{\parallel}] |\Upsilon_{\parallel}(h)|^2 w_{\parallel}(h) \\ &+ [\mathbf{m}(h)_{\perp}^* \cdot \mathbf{m}(h)_{\perp}] |\Upsilon_{\perp}(h)|^2 w_{\perp}(h) \}, \end{aligned} \quad (\text{A12})$$

and with

$$\mu_1 n_1 P_f = \omega \frac{(n_1 k_o)^3}{12\pi \varepsilon_o \varepsilon_1} |\alpha|^2 E_o^2, \quad (\text{A13})$$

the right-hand side of Eq. (A12) is the same as the emitted power in Eq. (68). So, the power emitted by the dipole is the same as the absorbed power from the laser beam. Here we only used Eq. (45) for $\bar{V}_p$, which is the same as Eq. (44) for α . Clearly, Eq. (44) is a necessary restriction on the polarizability of a particle if no energy is accumulating in the particle.

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Data Availability. No data were generated or analyzed in the presented research.

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