Journal of the Optical Society of America A
OPTICS, IMAGE SCIENCE, AND VISION

Propagation of magnetic dipole radiation through a medium

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Received 29 January 2016; revised 31 March 2016; accepted 31 March 2016; posted 1 April 2016 (Doc. ID 258121); published 15 April 2016

An oscillating magnetic dipole moment emits radiation. We assume that the dipole is embedded in a medium with relative permittivity ε_r and relative permeability μ_r, and we have studied the effects of the surrounding material on the flow lines of the emitted energy. For a linear dipole moment in free space the flow lines of energy are straight lines, coming out of the dipole. When located in a medium, these field lines curve toward the dipole axis, due to the imaginary part of μ_r. Some field lines end on the dipole axis, giving a nonradiating contribution to the energy flow. For a rotating dipole moment in free space, each field line of energy flow lies on a cone around the axis perpendicular to the plane of rotation of the dipole moment. The field line pattern is an optical vortex. When embedded in a material, the cone shape of the vortex becomes a funnel shape, and the windings are much less dense than for the pattern in free space. This is again due to the imaginary part of μ_r. When the real part of μ_r is negative, the field lines of the vortex swirl around the dipole axis opposite to the rotation direction of the dipole moment. For a near-single-negative medium, the spatial extent of the vortex becomes huge. We compare the results for the magnetic dipole to the case of an embedded electric dipole.

OCIS codes: (260.2110) Electromagnetic optics; (260.2160) Energy transfer; (350.5500) Propagation; (350.5610) Radiation.

http://dx.doi.org/10.1364/JOSAA.33.000882

1. INTRODUCTION

The optical properties of a linear, homogeneous, isotropic material are represented by the relative permittivity ε_r and the relative permeability μ_r. Both parameters are complex, in general, with a nonnegative imaginary part. The index of refraction n is defined as

$$n^2 = \varepsilon_r \mu_r, \qquad \mathrm{Im}\, n \geq 0. \tag{1}$$

This leaves an ambiguity if ε_r and μ_r are both positive or both negative. Then we include small positive imaginary parts in these parameters, and consider the limit where these imaginary parts approach zero. We then find that n is positive when ε_r and μ_r are both positive (normal dielectric material) and n is negative when ε_r and μ_r are both negative (negative index of refraction material). When a plane wave of light travels through a medium, the wavelength changes, as compared to propagation in free space, and this is due to the real part of n. The effect of the imaginary part of n is damping of the amplitudes of the electric and magnetic fields in the propagation direction, and this gives a corresponding damping of the intensity along the direction of propagation. The disappearing energy is absorbed by the material. The flow lines of energy are the field lines of the Poynting vector. For propagation in free space, these field lines are straight, and they remain straight for propagation in a material. One could argue that the damping only affects the magnitude of the Poynting vector, and, since field lines are only determined by the direction of the Poynting vector, the field lines should remain unaltered for propagation in a medium. The damping, due to absorption by the material, diminishes the intensity along the propagation direction, but it does not affect the paths of energy flow. For a plane wave, this argument holds true, but in general the effect of damping is more intricate, as we shall show below.

2. MAGNETIC DIPOLE RADIATION

We consider a magnetic dipole oscillating at angular frequency ω and located at the origin of coordinates. The dipole moment is given by

$$\mathbf{p}(t) = \mathrm{Re}(\mathbf{p}e^{-i\omega t}), \tag{2}$$

with $\mathbf{p}$ being the complex amplitude. The dipole is embedded in a medium with relative permittivity ε_r and relative permeability μ_r. The emitted electric field is time harmonic,

$$\mathbf{E}(\mathbf{r}, t) = \mathrm{Re}[\mathbf{E}(\mathbf{r})e^{-i\omega t}], \tag{3}$$

with $\mathbf{E}(\mathbf{r})$ being the complex amplitude, and a similar representation holds for the magnetic field $\mathbf{B}(\mathbf{r}, t)$. With a slight generalization of [1] we obtain,

$$\mathbf{E}(\mathbf{r}) = \frac{\mu_r n k_o^2}{4\pi\varepsilon_o c}(\mathbf{p}\times\hat{\mathbf{r}})\left(1+\frac{i}{nk_o r}\right)\frac{e^{ink_o r}}{r}, \tag{4}$$

$$\mathbf{B}(\mathbf{r}) = \frac{2\mu_o\mu_r}{3}\delta(\mathbf{r})\mathbf{p} + \frac{n}{c}\frac{\mu_r n k_o^2}{4\pi\varepsilon_o c}\{\mathbf{p}-(\mathbf{p}\cdot\hat{\mathbf{r}})\hat{\mathbf{r}} + [\mathbf{p}-3(\mathbf{p}\cdot\hat{\mathbf{r}})\hat{\mathbf{r}}]\}\frac{i}{nk_o r}\left(1+\frac{i}{nk_o r}\right)\frac{e^{ink_o r}}{r}, \tag{5}$$

with $k_o = \omega/c$ and $\hat{\mathbf{r}} = \mathbf{r}/r$.

In order to simplify the notation, we set $\mathbf{p} = p_o\hat{\mathbf{u}}$, with $p_o > 0$ and $\hat{\mathbf{u}}$ normalized as $\hat{\mathbf{u}}\cdot\hat{\mathbf{u}}^* = 1$. The overall constant is abbreviated as

$$\varsigma = \frac{\mu_r n k_o^3 p_o}{4\pi\varepsilon_o c}. \tag{6}$$

The dimensionless distance between the dipole and the field point is indicated by $q = k_o r$ and the dimensionless fields $\mathbf{e}(\mathbf{q})$ and $\mathbf{b}(\mathbf{q})$, with $\mathbf{q} = k_o\mathbf{r}$ are defined as

$$\mathbf{E}(\mathbf{r}) = \varsigma\mathbf{e}(\mathbf{q}), \tag{7}$$

$$\mathbf{B}(\mathbf{r}) = \frac{\varsigma}{c}\mathbf{b}(\mathbf{q}). \tag{8}$$

The self field on the right-hand side of Eq. (5) is irrelevant here, so we set $\mathbf{r}\neq 0$ from now on. We then obtain

$$\mathbf{e}(\mathbf{q}) = (\hat{\mathbf{u}}\times\hat{\mathbf{q}})\left(1+\frac{i}{nq}\right)\frac{e^{inq}}{q}, \tag{9}$$

$$\mathbf{b}(\mathbf{q}) = n\left\{\hat{\mathbf{u}}-(\hat{\mathbf{u}}\cdot\hat{\mathbf{q}})\hat{\mathbf{q}} + [\hat{\mathbf{u}}-3(\hat{\mathbf{u}}\cdot\hat{\mathbf{q}})\hat{\mathbf{q}}]\frac{i}{nq}\left(1+\frac{i}{nq}\right)\right\}\frac{e^{inq}}{q}. \tag{10}$$

Here, $\hat{\mathbf{q}} = \mathbf{q}/q = \hat{\mathbf{r}}$. These dimensionless fields only depend on the dimensionless position vector $\mathbf{q}$ of the field point.

3. POYNTING VECTOR

The time-averaged Poynting vector for electromagnetic radiation in a linear, homogeneous, isotropic material is given by

$$\mathbf{S}(\mathbf{r}) = \frac{1}{2\mu_o}\mathrm{Re}\left[\frac{1}{\mu_r}\mathbf{E}(\mathbf{r})^*\times\mathbf{B}(\mathbf{r})\right]. \tag{11}$$

For magnetic dipole radiation this can be expressed in terms of the dimensionless fields as

$$\mathbf{S}(\mathbf{q}) = \frac{|\varsigma|^2}{2\mu_o c}\mathrm{Re}\left[\frac{1}{\mu_r}\mathbf{e}(\mathbf{q})^*\times\mathbf{b}(\mathbf{q})\right]. \tag{12}$$

We notice that $\mathbf{S}$ only depends on the field point $\mathbf{r}$ through the dimensionless position vector $\mathbf{q}$.

With expressions (9) and (10), the right-hand side of Eq. (12) can be worked out. We split off an overall factor:

$$\mathbf{S}(\mathbf{q}) = \frac{|\varsigma|^2}{2\mu_o c}\frac{1}{q^2}e^{-2q\,\mathrm{Im}(n)}\boldsymbol{\sigma}(\mathbf{q}). \tag{13}$$

This defines the dimensionless vector $\boldsymbol{\sigma}(\mathbf{q})$. The positive overall factor depends on the field point through q. Since field lines of a vector field only depend on the direction of the vectors, the vector field $\boldsymbol{\sigma}(\mathbf{q})$ has the same field lines as the vector field $\mathbf{S}(\mathbf{q})$. We shall refer to $\boldsymbol{\sigma}(\mathbf{q})$ as the Poynting vector.

We obtain explicitly

$$\boldsymbol{\sigma}(\mathbf{q}) = [1-(\hat{\mathbf{u}}\cdot\hat{\mathbf{q}})(\hat{\mathbf{u}}^*\cdot\hat{\mathbf{q}})]\hat{\mathbf{q}}\,\mathrm{Re}\left[\frac{n}{\mu_r}\left(1+\frac{i}{nq}\right)^*\right] + \frac{1}{q|\mu_r|^2}\left|1+\frac{i}{nq}\right|^2\{[1-3(\hat{\mathbf{u}}\cdot\hat{\mathbf{q}})(\hat{\mathbf{u}}^*\cdot\hat{\mathbf{q}})]\hat{\mathbf{q}}\,\mathrm{Im}(\mu_r) + 2\,\mathrm{Im}[\mu_r(\hat{\mathbf{u}}^*\cdot\hat{\mathbf{q}})\hat{\mathbf{u}}]\}. \tag{14}$$

In terms of the dimensionless coordinate q, a distance of 2π corresponds to one free-space optical wavelength. The far field (many wavelengths from the source) is therefore the region $q \gg 1$. Equation (14) then simplifies to

$$\boldsymbol{\sigma}(\mathbf{q}) \approx [1-(\hat{\mathbf{u}}\cdot\hat{\mathbf{q}})(\hat{\mathbf{u}}^*\cdot\hat{\mathbf{q}})]\hat{\mathbf{q}}\,\mathrm{Re}\left(\frac{n}{\mu_r}\right). \tag{15}$$

The factor $1-(\hat{\mathbf{u}}\cdot\hat{\mathbf{q}})(\hat{\mathbf{u}}^*\cdot\hat{\mathbf{q}})$ is positive (except maybe zero for a certain direction $\hat{\mathbf{q}}$), and it can be shown that [2]

$$\mathrm{Re}\left(\frac{n}{\mu_r}\right) \geq 0. \tag{16}$$

The equal sign only holds for $\varepsilon_r > 0$ and $\mu_r < 0$, or $\varepsilon_r < 0$ and $\mu_r > 0$. Such materials are called single-negative, and we shall exclude this case for now. We come back to this interesting case in Section 6. The Poynting vector $\boldsymbol{\sigma}(\mathbf{q})$ in the far field is therefore approximately a positive (or zero) constant times $\hat{\mathbf{q}}$, and consequently the field lines are approximately straight, running outward from the dipole. Conversely, this implies that any curving of the field lines can only occur in the near field.

Let us now return to the general expression (14). It can be shown that

$$\mathrm{Re}\left[\frac{n}{\mu_r}\left(1+\frac{i}{nq}\right)^*\right] \geq 0, \tag{17}$$

with the equal sign only holding for single-negative materials. Therefore, the first term on the right-hand side of Eq. (14) is a positive (or zero) constant times $\hat{\mathbf{q}}$, giving rise to radially outward-running straight field lines. The first term in braces on the right-hand side of Eq. (14) is also proportional to $\hat{\mathbf{q}}$ (although the multiplying factor may not be positive), so this term also gives a radial contribution to the field lines. The second term in braces, $2\,\mathrm{Im}[\mu_r(\hat{\mathbf{u}}^*\cdot\hat{\mathbf{q}})\hat{\mathbf{u}}]$, is not proportional to $\hat{\mathbf{q}}$, and therefore any curving of the field lines comes from this term.

It is interesting to notice that the material parameters in the expressions above only appear through μ_r and n. There is no explicit dependence on ε_r. Equation (14) in [3] gives the expression for $\boldsymbol{\sigma}(\mathbf{q})$ for the case of an embedded electric dipole. The terms in braces are identical, with ε_r and μ_r switched. For an electric dipole, however, there is also an explicit dependence on μ_r.

The field lines of the Poynting vector are obtained as follows. Let $\mathbf{q}(u)$ be a parametrization of a field line. Any field line is the solution of $\mathrm{d}\mathbf{q}(u)/\mathrm{d}u = \boldsymbol{\sigma}(\mathbf{q}(u))$. We select an initial point with Cartesian dimensionless coordinates $(\bar{x}_o, \bar{y}_o, \bar{z}_o)$. Here, $\bar{x} = k_o x$, and so on. The field line through the selected point follows from integrating $\mathrm{d}\mathbf{q}(u)/\mathrm{d}u = \boldsymbol{\sigma}(\mathbf{q}(u))$, with $u = 0$ corresponding to the initial point. The direction of the field line is the direction of increasing u. The numerical integration is done with Mathematica.

4. LINEAR DIPOLE

When the unit vector $\hat{\mathbf{u}}$ is real, we have $\mathbf{p}(t) = p_o \hat{\mathbf{u}} \cos(\omega t)$, and the dipole moment oscillates along the vector $\hat{\mathbf{u}}$. This is a linear dipole. We shall take $\hat{\mathbf{u}} = \mathbf{e}_z$, so the dipole moment oscillates along the z axis. It is easy to verify that $\boldsymbol{\sigma}(\mathbf{q})$ is rotation symmetric around the z axis and reflection symmetric in the xy plane. We therefore only consider field lines in the yz plane, with $y \geq 0$ and $z \geq 0$. We have $\hat{\mathbf{u}} \cdot \hat{\mathbf{q}} = \cos(\theta)$. The term $2\,\mathrm{Im}[\mu_r(\hat{\mathbf{u}}^* \cdot \hat{\mathbf{q}})\hat{\mathbf{u}}]$ becomes $2\mathbf{e}_z \cos(\theta)\mathrm{Im}(\mu_r)$. This term is proportional to $\mathbf{e}_z$, whereas all other terms are proportional to $\hat{\mathbf{q}}$. Moreover, this term is positive, and therefore the field lines will deviate from the radial direction such that they bend "up" toward the positive z axis. Furthermore, the term is proportional to $\mathrm{Im}(\mu_r)$, and therefore the curving is entirely due to the imaginary part of the permeability.

We find explicitly,

$$\boldsymbol{\sigma}(\mathbf{q}) = \sin^2(\theta)\hat{\mathbf{q}}\,\mathrm{Re}\left[\frac{n}{\mu_r}\left(1 + \frac{i}{nq}\right)^*\right] + \frac{1}{q|\mu_r|^2}\left|1 + \frac{i}{nq}\right|^2 \{[1 - 3\cos^2(\theta)]\hat{\mathbf{q}} + 2\mathbf{e}_z \cos(\theta)\}\mathrm{Im}(\mu_r). \quad \textbf{(18)}$$

Interestingly, the entire term in braces is proportional to $\mathrm{Im}(\mu_r)$. For free space we have $\varepsilon_r = \mu_r = n = 1$, and the Poynting vector becomes $\boldsymbol{\sigma}(\mathbf{q}) = \sin^2(\theta)\hat{\mathbf{q}}$. The field line pattern is shown in Fig. 1. The field lines are straight, coming out of the dipole. Figure 2 shows the field lines for $\varepsilon_r = 2$ and $\mu_r = 1.5 + 0.8i$. The field lines bend up due to $\mathrm{Im}(\mu_r) \neq 0$. We see from the figure that near the z axis the field lines end on the z axis rather than running to infinity. Consider a field point close to the z axis, so $\bar{y}$ is small. Equation (18) becomes approximately

$$\boldsymbol{\sigma}(\mathbf{q}) \approx -2\mathbf{e}_y \frac{\bar{y}}{q^2|\mu_r|^2}\left|1 + \frac{i}{nq}\right|^2 \mathrm{Im}(\mu_r). \quad \textbf{(19)}$$

We see that the Poynting vector is into the negative y direction, and therefore every field line hits the z axis under 90°. The field lines end at the z axis. The z axis is a singular line, and $\boldsymbol{\sigma}(\mathbf{q}) = 0$ on the z axis. Energy flowing along field lines that end at the z axis does not contribute to the radiated power. Similar behavior was found for an electric dipole [4], where the bending of the field lines resulted from the imaginary part of the permittivity. Figure 3 shows a larger view of the same graph as in Fig. 2. We see that there is a cylindrical subwavelength region around the z axis that contains the field lines that end up on the z axis. Field lines outside this region run to infinity, and they level off to straight lines in the far field.

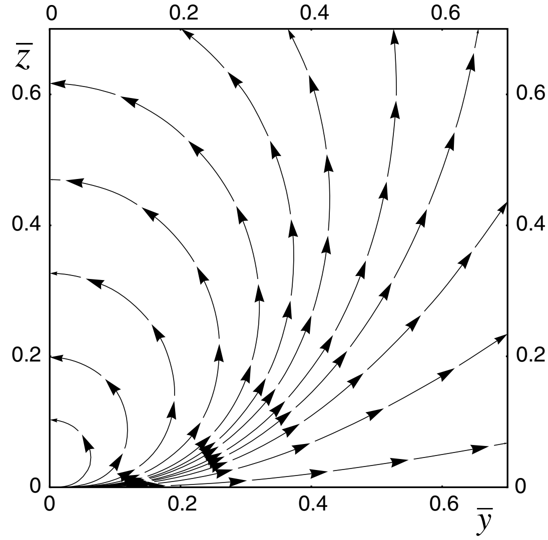

Fig. 2. Field lines for a linear magnetic dipole embedded in a medium with $\varepsilon_r = 2$ and $\mu_r = 1.5 + 0.8i$ are curved. The bending toward the z axis is a consequence of the nonzero imaginary part of μ_r.

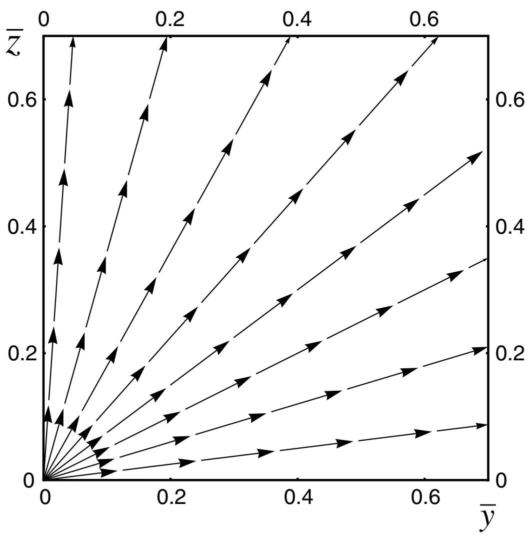

Fig. 1. Figure shows the field lines of the Poynting vector for a magnetic dipole oscillating along the z axis. The field lines are straight. The shown density of the field lines has no significance.

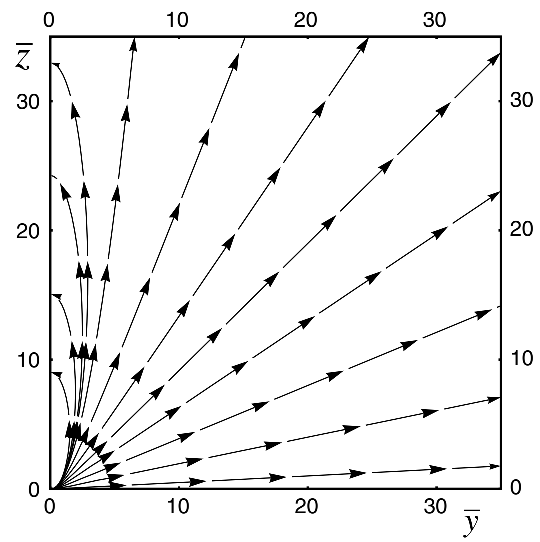

Fig. 3. Figure shows a larger view of the field line pattern of Fig. 2. Field lines close to the z axis bend toward the axis, hitting it perpendicularly. Other field lines run to infinity as almost straight lines.

5. CIRCULAR DIPOLE

When we take vector $\hat{\mathbf{u}}$ as

$$\hat{\mathbf{u}} = -\frac{1}{\sqrt{2}}(\mathbf{e}_x + i\mathbf{e}_y), \tag{20}$$

then the dipole moment $\mathbf{p}(t)$ rotates in the xy plane with angular frequency ω. The rotation direction is counterclockwise when viewed down the positive z axis. We now have

$$\hat{\mathbf{u}} \cdot \hat{\mathbf{q}} = -\frac{1}{\sqrt{2}} \sin(\theta)e^{i\phi}, \tag{21}$$

and the only term which is not proportional to $\hat{\mathbf{q}}$ becomes

$$2\,\mathrm{Im}[\mu_r(\hat{\mathbf{u}}^* \cdot \hat{\mathbf{q}})\hat{\mathbf{u}}] = \sin(\theta)[\mathbf{e}_\rho \,\mathrm{Im}(\mu_r) + \mathbf{e}_\phi \,\mathrm{Re}(\mu_r)]. \tag{22}$$

The unit vectors $\mathbf{e}_\rho$ and $\mathbf{e}_\phi$ are given by

$$\mathbf{e}_\rho = \mathbf{e}_x \cos(\phi) + \mathbf{e}_y \sin(\phi), \tag{23}$$

$$\mathbf{e}_\phi = -\mathbf{e}_x \sin(\phi) + \mathbf{e}_y \cos(\phi), \tag{24}$$

and therefore the right-hand side of Eq. (22) is a vector in the xy plane. The Poynting vector from Eq. (14) becomes

$$\begin{aligned}\boldsymbol{\sigma}(\mathbf{q}) = {} & \left[1 - \frac{1}{2}\sin^2(\theta)\right]\hat{\mathbf{q}}\,\mathrm{Re}\left[\frac{n}{\mu_r}\left(1 + \frac{i}{nq}\right)^*\right] \\ & + \frac{1}{q|\mu_r|^2}\left|1 + \frac{i}{nq}\right|^2\left\{\left[1 - \frac{3}{2}\sin^2(\theta)\right]\hat{\mathbf{q}}\,\mathrm{Im}(\mu_r) \right. \\ & \left. + \sin(\theta)[\mathbf{e}_\rho \,\mathrm{Im}(\mu_r) + \mathbf{e}_\phi \,\mathrm{Re}(\mu_r)]\right\} \ldots. \end{aligned} \tag{25}$$

The Poynting vector is reflection symmetric in the xy plane, so we consider $z \geq 0$ only.

For a circular magnetic dipole in free space we have

$$\boldsymbol{\sigma}(\mathbf{q}) = \left[1 - \frac{1}{2}\sin^2(\theta)\right]\hat{\mathbf{q}} + \frac{1}{q}\left(1 + \frac{1}{q^2}\right)\sin(\theta)\mathbf{e}_\phi. \tag{26}$$

This result is identical to the expression for an electric dipole in free space [5]. The basis vectors in a spherical coordinate system are $\hat{\mathbf{q}}$, $\mathbf{e}_\theta$, and $\mathbf{e}_\phi$. Vector $\boldsymbol{\sigma}(\mathbf{q})$ has no $\mathbf{e}_\theta$ component, and therefore θ is constant along a field line. Each field line lies on a cone around the z axis. The $\mathbf{e}_\phi$ component is positive, and therefore the field lines wind around the z axis into the direction of increasing ϕ. This is counterclockwise when viewed down the positive z axis, and so the rotation direction of the field lines around the z axis is the same as the rotation direction of the rotating dipole moment. Figure 4 shows a typical field line. Within about a wavelength from the location of the dipole, a field line winds around the z axis, and at a larger distance it levels off to approximately a straight line. The field line pattern has a vortex structure near the dipole, and the field lines run off in the radial direction in the far field. For a point on the positive z axis we have $\theta = 0$, and the Poynting vector becomes $\boldsymbol{\sigma}(\mathbf{q}) = \mathbf{e}_z$. Therefore, the z axis is a field line.

For radiation in a medium, the Poynting vector is given by Eq. (25). It is easy to see that the z axis is still a field line. Off the z axis, the Poynting vector has a $\hat{\mathbf{q}}$ component and an $\mathbf{e}_\phi$ component. Vector $\mathbf{e}_\rho$ can be expressed as

$$\mathbf{e}_\rho = \hat{\mathbf{q}} \sin(\theta) + \mathbf{e}_\theta \cos(\theta), \tag{27}$$

so vector $\boldsymbol{\sigma}(\mathbf{q})$ also has an $\mathbf{e}_\theta$ component. Therefore θ varies along a field line. The $\mathbf{e}_\theta$ component equals $\sin(\theta)\cos(\theta)$

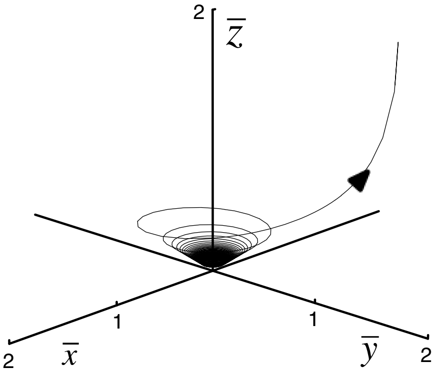

Fig. 4. Shown is a field line of the Poynting vector for a rotating dipole moment in free space. The field line lies on a cone around the z axis. Close to the dipole, it winds around the z axis numerous times, and in the far field it levels off to approximately a straight line.

$\mathrm{Im}(\mu_r)$ multiplied by a positive function of q. For $z > 0$ this is positive, provided that $\mathrm{Im}(\mu_r) \neq 0$. Therefore, θ increases along a field line, and the cone shape for radiation in free space becomes a funnel shape. This is shown in Fig. 5. The funnel shape is due to the imaginary part of μ_r. For an electric dipole, a funnel shape appears due to the imaginary part of ε_r. Another effect is that with an increasing of $\mathrm{Im}(\mu_r)$, the windings around the z axis become less dense. This is illustrated in Fig. 6.

The swirling of the field lines around the z axis comes from the $\mathbf{e}_\phi$ component of $\boldsymbol{\sigma}(\mathbf{q})$. This component is a positive function of q times $\mathrm{Re}(\mu_r)$. If $\mathrm{Re}(\mu_r) > 0$, as for free space, the rotation direction is counterclockwise when viewed down the positive z axis, and this is the same rotation direction as the magnetic dipole moment. If $\mathrm{Re}(\mu_r) < 0$, however, the $\mathbf{e}_\phi$ component of $\boldsymbol{\sigma}(\mathbf{q})$ is negative, and this reverses the rotation direction of the field lines around the z axis. In this case, the flow of energy counter-rotates the rotation direction of the dipole moment. For an electric dipole, the same effects are attributed to $\mathrm{Re}(\varepsilon_r)$.

6. NEAR-SINGLE-NEGATIVE MATERIALS

For a circular dipole we have in the far field,

$$\boldsymbol{\sigma}(\mathbf{q}) \approx \left[1 - \frac{1}{2}\sin^2(\theta)\right]\hat{\mathbf{q}}\,\mathrm{Re}\left[\frac{n}{\mu_r}\left(1 + \frac{i}{nq}\right)^*\right], \tag{28}$$

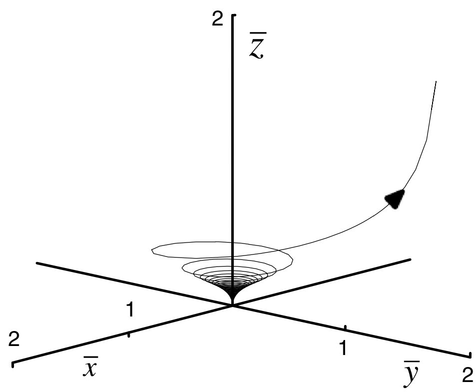

Fig. 5. Graph shows a field line of the Poynting vector for material parameters $\varepsilon_r = 1$ and $\mu_r = 0.8 + 0.01i$. The cone shape of Fig. 4 becomes a funnel shape due to $\mathrm{Im}(\mu_r) \neq 0$.

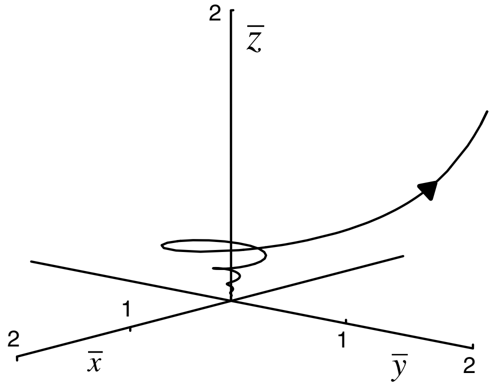

Fig. 6. Shown is a field line of the Poynting vector for $\varepsilon_r = 1$ and $\mu_r = 0.8 + 0.1i$. The only difference with the parameters for Fig. 5 is that the imaginary part of μ_r is 10 times larger. As a result, the windings are much less tight.

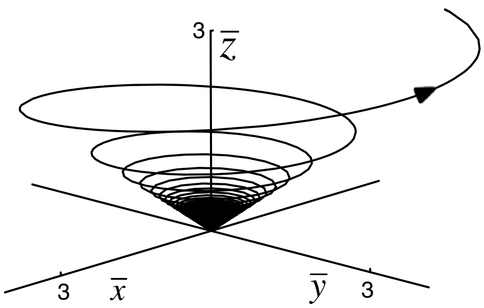

Fig. 7. Shown is a field line of the Poynting vector for a rotating dipole. The material parameters are $\varepsilon_r = -1 + 0.1i$ and $\mu_r = 0.8$.

which is proportional to $\hat{\mathbf{q}}$. Therefore, the field lines run approximately radially outward, and they are approximately straight. A single-negative material has a real ε_r and a real μ_r, and they are of opposite sign. From Eq. (1) we see that n^2 is negative, and so n is positive imaginary. The expression in square brackets in Eq. (28) is pure imaginary, so $\mathrm{Re}[\ldots] = 0$. We conclude that for a single-negative material the right-hand side of Eq. (28) vanishes. Apparently, for such materials the far-field term of the field line pattern is absent. Consequently, the remaining near-field terms continue to dominate in the far field. In the near field, the field lines swirl around the z axis, and therefore we expect this pattern to extend into the far field. The vortex structure is not of subwavelength scale anymore, but extends into the far field. This leads to a huge vortex pattern, when seen on the scale of a wavelength.

A perfect single-negative material does not exist, so we consider near-single-negative materials, which have a small positive imaginary part in ε_r, in μ_r, or in both. Figure 7 shows a field line of the Poynting vector for $\varepsilon_r = -1 + 0.1i$ and $\mu_r = 0.8$. The field line lies on a cone, because $\mathrm{Im}(\mu_r) = 0$. As compared

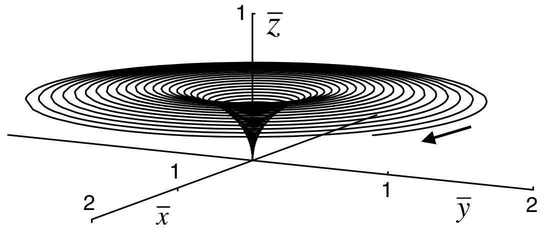

Fig. 8. Shown is a field line of the Poynting vector for a rotating dipole. The material parameters are $\varepsilon_r = 1$ and $\mu_r = -0.8 + 0.01i$.

to Fig. 4, we see that the spatial extent of the vortex is very large. If we make $\mathrm{Im}(\varepsilon_r)$ smaller, the size of the vortex increases even more. Figure 8 shows a field line for $\varepsilon_r = 1$ and $\mu_r = -0.8 + 0.01i$. The shape is a funnel; the rotation direction is reversed, as compared to the rotation direction of the dipole moment; and the spatial extent of the vortex is immense. In fact, it is much larger than shown in the figure. Also interesting to see is that the field line flattens out. At some distance from the dipole, the field line appears to be rotating around the z axis while remaining approximately in a plane parallel to the xy plane. In the theoretical limit of a perfect single-negative material, the field lines are circles around the z axis.

7. CONCLUSIONS

A magnetic dipole is embedded in a medium with relative permittivity ε_r and relative permeability μ_r. We have studied the energy flow patterns of the emitted radiation as it propagates through the material. For a linear dipole, the field lines of energy flow bend toward the dipole axis due to the imaginary part of μ_r. Field lines close to the axis end at the axis, whereas other field lines run to infinity. For a circular dipole, the field lines wind around the axis perpendicular to the plane of rotation in the near field. In the far field they level off to straight lines. In free space, each field line lies on a cone. The effect of the imaginary part of μ_r is that the cone shape becomes a funnel shape, and the windings are less dense than for the case of free space. When the real part of μ_r is negative, the rotation of the field lines around the axis is opposite to the rotation direction of the magnetic dipole moment. For a near-single-negative embedding medium, the spatial extent of the optical vortex becomes enormous.

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