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# APPLICATION OF THE MAGNETIC FIELD INTEGRAL EQUATION TO DIFFRACTION AND REFLECTION BY A CONDUCTING SHEET

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**Abstract**

The Magnetic Field Integral Equation is an integral equation for the current density induced on the surface of a perfect conductor by an incident electromagnetic field. For many scattering problems this equation can be solved for the current density, after which the scattered field can be obtained by integration. For scattering off a mirror, the Magnetic Field Integral Equation can be solved easily. The current density at a point on the mirror is determined in a simple way by the incident magnetic field at the same point. We show that for reflection and diffraction of electromagnetic radiation by a thin sheet of finite size and possibly with an aperture, application of the Magnetic Field Integral Equation yields a relation between the current densities at both sides of the thin sheet (illuminated and shadow sides). The resulting equation relates the current densities at opposites sides of the sheet, evaluated at the same point, to the incident magnetic field at that point. For an infinite sheet without an aperture the equation reduces to the result for a mirror.

## 1. Introduction

When electromagnetic radiation is incident upon a dielectric or metallic object, it induces a current density in the material. This current density generates electromagnetic radiation, which is observed as the scattered, diffracted or reflected field by the object. Inside the medium, this radiation from the current density adds to the incident field, giving the refracted field. One approach for solving several such scattering problems is the boundary condition method, which is also referred to as the differential equation method. For the material, a constitutive equation relates the current density to the local electric field. Maxwell's equations for the electric and magnetic fields then become homogeneous, and general solutions of these equations are constructed for the region

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inside and the region outside the object. The arbitrary constants in the solution are then determined by applying the boundary conditions for the interface of the two media. This method has been proven successful for scattering of plane waves by a sphere or small particles [1-6]. An entirely different approach is the integral equation method. By manipulating Maxwell's equations, an integral equation for the electric or magnetic field is derived. The first integral equation appears to be Kirchhoff's integral theorem [7,8], which expresses the electric field at a given point in terms of its values and normal derivatives everywhere on a boundary surface. The Ewald-Oseen extinction theorem and variations on this theorem [9-14] also involve the values and the normal derivatives of the fields at the boundary, and these are assumed to be known.

Kirchhoff's integral theorem and the various extinction theorems follow from applications of Green's theorem. In an alternative approach, the electric field is considered as generated by a current density in the material. The diffracted or reflected electric field can then be written as an integral over the unknown current density, involving the Green's function for the wave equation. An additional constitutive equation then relates the current density to the electric field, yielding an integral equation for the electric field which has the incident electric field as the inhomogeneous term. In a similar way an equation for the magnetic field can be derived, and there exist all kinds of variations on integral equations of this type [15,16]. A major advantage over relations derived from Green's theorem is that these integral equations do not require the knowledge of the fields and their normal derivatives on the boundary. Another advantage is that any solution automatically satisfies the boundary conditions at the interface [17].

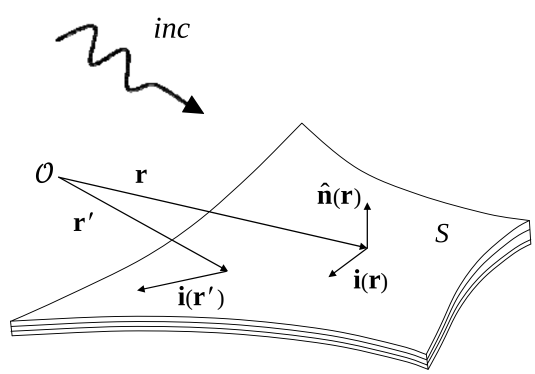


Figure 1. An electromagnetic field is incident upon a metallic object, assumed to have infinite conductivity. The surface $S$ of the material can be any shape, and either infinite in extend or closed. The unit normal vector $\hat{\mathbf{n}}(\mathbf{r})$ at point $\mathbf{r}$ is directed from the medium towards free space, and the origin of coordinates $\mathcal{O}$ can be chosen arbitrarily. The Magnetic Field Integral Equation relates the surface current density $\mathbf{i}(\mathbf{r})$ to the values $\mathbf{i}(\mathbf{r}')$ at all other points $\mathbf{r}'$.

Particularly interesting is the diffraction and reflection of radiation by a metallic object in the limit where the conductivity of the material is infinite. Such material is impenetrable for radiation, and therefore all induced current is confined to the surface of the material. This surface current density $\mathbf{i}(\mathbf{r},t)$ generates the scattered field, and inside the perfect conductor the field produced by $\mathbf{i}(\mathbf{r},t)$ cancels exactly the incident field. We shall assume a harmonic time dependence with angular frequency $\omega$ for the incident field. The magnetic component of this field will be written as

$$\mathbf{B}(\mathbf{r},t)_{\mathrm{inc}} = \mathrm{Re}\left(\mathbf{B}(\mathbf{r})_{\mathrm{inc}}\, e^{-i\omega t}\right) , \tag{1}$$

with $\mathbf{B}(\mathbf{r})_{\mathrm{inc}}$ the complex amplitude, and all time dependent fields will have the same time dependence, in particular $\mathbf{i}(\mathbf{r},t)$. The incident field illuminates the material, as illustrated in Figure 1, and a current density $\mathbf{i}(\mathbf{r})$ is induced in the surface at the point **r**. For this situation, an integral equation for $\mathbf{i}(\mathbf{r})$ can be derived, which has $\mathbf{B}(\mathbf{r})_{\mathrm{inc}}$ as inhomogeneous term. This integral equation (next section) is known as the Magnetic Field Integral Equation, although it is an equation for the surface current density. This equation has been widely applied to numerically solve scattering problems for objects of arbitrary shape and to the computation of the field reflected by a rough surface [18,19]. We shall consider the reflection and diffraction of incident radiation by a thin sheet with an aperture and of finite dimension. The current density $\mathbf{i}(\mathbf{r})$ generates specular (reflected) radiation, and leads to diffraction around the edges. We shall show that application of the Magnetic Field Integral Equation on one hand yields an interesting result for the current density, but on the other hand does not provide a complete account of the scattering problem.

## 2. The Magnetic Field Integral Equation

Let an electromagnetic field with angular frequency $\omega$ be incident upon an object with a surface $S$ of arbitrary shape, as in Figure 1. For perfectly-conducting material, all induced current is surface current $\mathbf{i}(\mathbf{r})$. The magnetic field $\mathbf{B}(\mathbf{r})$ at point **r** can then be written as

$$\mathbf{B}(\mathbf{r}) = \mathbf{B}(\mathbf{r})_{\mathrm{inc}} - \frac{\mu_{\mathrm{o}}}{4\pi}\int \mathrm{d}S'\,\mathbf{i}(\mathbf{r}')\times\nabla g(\mathbf{r}-\mathbf{r}') \ , \quad (\mathbf{r}\text{ off } S) \tag{2}$$

with $\mathbf{B}(\mathbf{r})_{\mathrm{inc}}$ the given, but further arbitrary, incident field. The second term on the right-hand side equals the magnetic field generated by the current, with

$$g(\mathbf{r}-\mathbf{r}') = \frac{e^{ik_{\mathrm{o}}|\mathbf{r}-\mathbf{r}'|}}{|\mathbf{r}-\mathbf{r}'|} \ , \quad k_{\mathrm{o}} = \omega / c \ , \tag{3}$$

the free-space Green's function for the Helmholtz equation. The integral in Eq. (2) runs over the surface *S*, and **r**' indicates a point in *S*. It is essential that the field point **r** is not in *S*, since $\nabla g(\mathbf{r}-\mathbf{r}')$ has a singularity for $\mathbf{r}'\to\mathbf{r}$.

Let us now consider the field points $\mathbf{r}_+$ and $\mathbf{r}_-$, just outside and inside the material, respectively, and near the point **r** in *S*. With $\hat{\mathbf{n}}(\mathbf{r})$ the unit normal vector on *S* at **r**, directed from the medium into the vacuum, we can then write $\mathbf{r}_\pm = \mathbf{r}\pm\varepsilon\,\hat{\mathbf{n}}(\mathbf{r})$ with $\varepsilon$ small. As long as $\varepsilon$ is finite, Eq. (2) applies with **r** replaced by $\mathbf{r}_\pm$. For the integral over *S*, we leave out a circle with radius $\delta$ around **r**, and then let $\varepsilon$ approach zero. In the limit $\varepsilon\to 0$, there is a finite contribution from the singularity at $\mathbf{r}'=\mathbf{r}$, and this contribution remains finite for $\delta\to 0$. The result of this procedure is [17]

$$\mathbf{B}(\mathbf{r}_\pm) = \mathbf{B}(\mathbf{r})_{\mathrm{inc}} \pm \tfrac{1}{2}\mu_o\mathbf{i}(\mathbf{r})\times\hat{\mathbf{n}}(\mathbf{r}) - \frac{\mu_o}{4\pi}P\int \mathrm{d}S'\,\mathbf{i}(\mathbf{r}')\times\nabla g(\mathbf{r}-\mathbf{r}') \quad ,\ (\mathbf{r}\text{ in } S) \tag{4}$$

expressing the total magnetic field just outside and inside the material as the sum of the incident field at **r** and the contribution from the current density $\mathbf{i}(\mathbf{r})$. The integral over *S* is now a Cauchy Principal Value integral, and the second term on the right-hand side is the finite contribution from the singularity. Taking the difference between the plus and minus equations yields

$$\mathbf{B}(\mathbf{r}_+) - \mathbf{B}(\mathbf{r}_-) = \mu_o\mathbf{i}(\mathbf{r})\times\hat{\mathbf{n}}(\mathbf{r}) \ , \tag{5}$$

which is the usual boundary condition for an interface carrying a surface current density $\mathbf{i}(\mathbf{r})$.

When the material is a perfect conductor, the electromagnetic field inside vanishes. In particular at the point $\mathbf{r}_-$ we have $\mathbf{B}(\mathbf{r}_-)=0$ and Eq. (4) with the lower sign becomes

$$\mathbf{i}(\mathbf{r}) + \frac{1}{2\pi}\hat{\mathbf{n}}(\mathbf{r})\times P\int \mathrm{d}S'\,\mathbf{i}(\mathbf{r}')\times\nabla g(\mathbf{r}-\mathbf{r}') = \frac{2}{\mu_o}\hat{\mathbf{n}}(\mathbf{r})\times\mathbf{B}(\mathbf{r})_{\mathrm{inc}} \quad , \tag{6}$$

after taking the cross product with $\hat{\mathbf{n}}(\mathbf{r})$. This integral equation for the unknown current density $\mathbf{i}(\mathbf{r})$ is the Magnetic Field Integral Equation, originally due to Maue [20]. After solving Eq. (6) for $\mathbf{i}(\mathbf{r})$, if possible, the magnetic field at a field point **r** outside the material follows by integration from Eq. (2). For any **r** inside the medium, the term with the integral in Eq. (2) should cancel exactly the incident field $\mathbf{B}(\mathbf{r})_{\mathrm{inc}}$ when $\mathbf{i}(\mathbf{r})$ is a solution of Eq. (6). Equation (5) becomes $\mathbf{B}(\mathbf{r}_+) = \mu_o\mathbf{i}(\mathbf{r})\times\hat{\mathbf{n}}(\mathbf{r})$ for a perfect conductor, and the cross product with $\hat{\mathbf{n}}(\mathbf{r})$ gives

$$\mathbf{i}(\mathbf{r}) = \frac{1}{\mu_o}\hat{\mathbf{n}}(\mathbf{r})\times\mathbf{B}(\mathbf{r}_+) \ , \tag{7}$$

since $\mathbf{i}(\mathbf{r})$ is in the tangent plane of *S* at **r**.

## 3. Reflection off a Mirror

A mirror is an infinite slab of perfect conductor with thickness $\Delta L$, as shown in Figure 2. Since the surface of the mirror is flat, the unit normal vector $\hat{\mathbf{n}}(\mathbf{r})$ is the same for all $\mathbf{r}$, and points from the material to the vacuum. The gradient of the Green's function is

$$\nabla g(\mathbf{r}-\mathbf{r}') = \frac{\mathbf{r}-\mathbf{r}'}{|\mathbf{r}-\mathbf{r}'|^3}\left(ik_o\,|\mathbf{r}-\mathbf{r}'|-1\right)e^{ik_o|\mathbf{r}-\mathbf{r}'|} \;, \tag{8}$$

which is proportional to $\mathbf{r}-\mathbf{r}'$. In Eq. (6), both $\mathbf{r}$ and $\mathbf{r}'$ are in the surface of the mirror, and so is $\mathbf{i}(\mathbf{r}')$. Therefore we see that $\mathbf{i}(\mathbf{r}')\times\nabla g(\mathbf{r}-\mathbf{r}')$ is proportional to $\hat{\mathbf{n}}$, and we have

$$\hat{\mathbf{n}}\times[\mathbf{i}(\mathbf{r}')\times\nabla g(\mathbf{r}-\mathbf{r}')] = 0 \;. \tag{9}$$

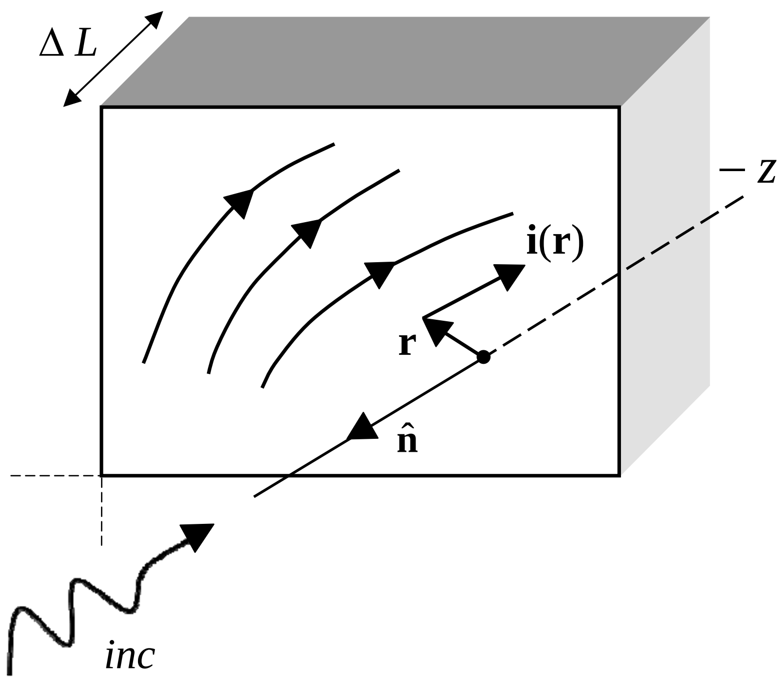


Figure 2. A mirror is a (flat) layer of perfectly-conducting material, infinite in extend, and with thickness $\Delta L$. The incident field induces a current density $\mathbf{i}(\mathbf{r})$ on the surface, and this vector field can be visualized by its field line pattern, as schematically indicated by the curves.

The Magnetic Field Integral Equation for a mirror thus reduces to [21]

$$\mathbf{i}(\mathbf{r}) = \frac{2}{\mu_o}\hat{\mathbf{n}} \times \mathbf{B}(\mathbf{r})_{\rm inc} \ , \tag{10}$$

which is also the solution $\mathbf{i}(\mathbf{r})$ of this equation. We find that the surface current density in a mirror at position $\mathbf{r}$ on its surface is determined by the incident magnetic field at the same position. It is interesting to compare this result to the boundary condition (7), which holds for any surface. In Eq. (7), the field $\mathbf{B}(\mathbf{r}_+)$ is the total magnetic field just outside the conductor, which includes the magnetic field generated by the current everywhere on the surface. The field $\mathbf{B}(\mathbf{r}_+)$ in Eq. (7) is unknown, whereas the field $\mathbf{B}(\mathbf{r})_{\rm inc}$ in Eq. (10) is given.

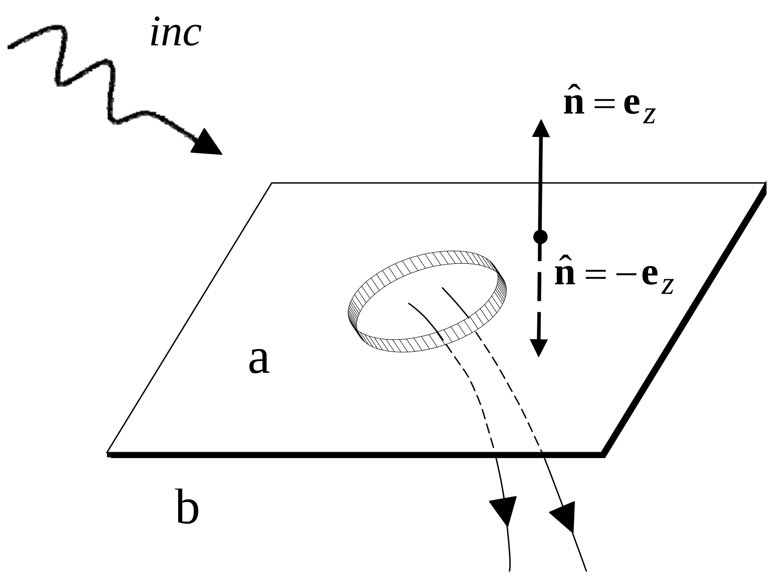


Figure 3. A field is incident upon a perfectly-conducting thin sheet of finite size, and possibly with an aperture. The *z*-direction is taken as in Figure 2. so that the unit normal vectors $\hat{\mathbf{n}}$ are $\mathbf{e}_z$ and $-\mathbf{e}_z$, for the illuminated and shadow sides, respectively

## 4. Scattering off a Thin Sheet

The current density on the surface of a mirror, given by Eq. (10), is independent of the thickness $\Delta L$ of the conductor, and holds therefore also in the limit $\Delta L \to 0$. We shall now consider a (flat) sheet of perfectly-conducting material with thickness $\Delta L$, as in Figure 3. The sheet may be finite in extend and may have an opening (aperture) in it. The

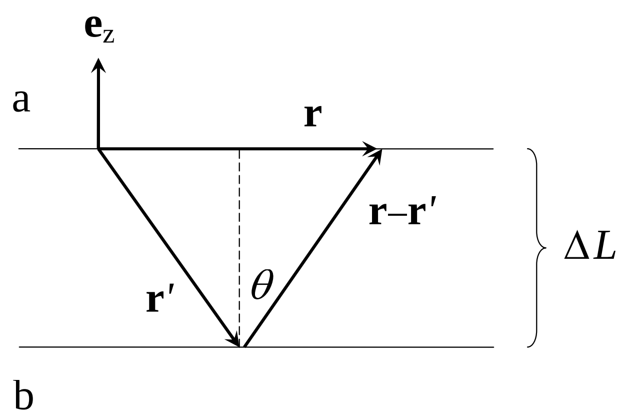


Figure 4. The figure illustrates the geometry for the contribution of the current density at point $\mathbf{r}'$ at the *b*-side to the current density at point $\mathbf{r}$ at the *a*-side in the Magnetic Field Integral Equation.

incident field will then pass through the aperture and continue past the edges. It will also diffract around the edges of the screen and the aperture. We shall call the illuminated side the *a* side and the shadow side the *b* side, as in the figure. For the mirror, all current density was at the *a* side, but now radiation can appear at the *b* side as well, and this will induce a current density at this shadow side. We shall indicate the respective current densities by $\mathbf{i}_\mathrm{a}$ and $\mathbf{i}_\mathrm{b}$, and consider the limit $\Delta L \to 0$. For a thin sheet without edges or an aperture (mirror) we have $\mathbf{i}_\mathrm{b} = 0$, and $\mathbf{i}_\mathrm{a}$ is given by Eq. (10).

Let the *a* side of the material be the *xy*-plane, with the unit normal for this surface equal to $\mathbf{e}_z$. Then the *b* side has the same shape as the *a* side, is displaced over a distance $\Delta L$ in the negative *z*-direction, and has $-\mathbf{e}_z$ as its unit normal. Let the fixed point $\mathbf{r}$ in the Magnetic Field Integral Equation be in the *a* side, so that the first term on the left-hand side of Eq. (6) is $\mathbf{i}_\mathrm{a}(\mathbf{r})$. The integral in Eq. (6) runs over both the *a*- and the *b*-surface, and for $\Delta L \to 0$ there is no contribution from the edges. For any point $\mathbf{r}'$ in surface *a*, the integrand vanishes for the same reason as in Sec. 3. Therefore, the integral in Eq. (6) only has a contribution from the *b*-surface. Then the *P* in front of the integral can be omitted, and with $\hat{\mathbf{n}}(\mathbf{r}) = \mathbf{e}_z$, a vector identity and $\mathbf{i}_\mathrm{b}(\mathbf{r}') \cdot \mathbf{e}_z = 0$, the theorem becomes

$$\mathbf{i}_\mathrm{a}(\mathbf{r}) + \frac{1}{2\pi} \int \mathrm{d}S' \, \mathbf{i}_\mathrm{b}(\mathbf{r}') [\mathbf{e}_z \cdot \nabla g(\mathbf{r}-\mathbf{r}')] = \frac{2}{\mu_\mathrm{o}} \mathbf{e}_z \times \mathbf{B}(\mathbf{r})_\mathrm{inc} \ , \qquad (11)$$

for $\Delta L \to 0$. Let us now consider the contribution from $\mathbf{i}_\mathrm{b}$ at $\mathbf{r}'$ for a given $\mathbf{r}$. When the sheet thickness $\Delta L$ approaches zero, angle $\theta$ in Figure 4 goes to $\pi/2$. Since $\nabla g(\mathbf{r}-\mathbf{r}')$ is proportional to $\mathbf{r}-\mathbf{r}'$ we see that $\mathbf{e}_z \cdot \nabla g(\mathbf{r}-\mathbf{r}')$ vanishes for $\Delta L \to 0$. Or with $\mathbf{e}_z \cdot (\mathbf{r}-\mathbf{r}') = \Delta L$ we have from Eq. (8)

$$\mathbf{e}_z \cdot \nabla g(\mathbf{r}-\mathbf{r}') = \Delta L \frac{1}{|\mathbf{r}-\mathbf{r}'|^3}\left(ik_o\,|\mathbf{r}-\mathbf{r}'|-1\right)e^{ik_o|\mathbf{r}-\mathbf{r}'|} \ , \tag{12}$$

for any $\Delta L$, and this goes to zero for $\Delta L \to 0$. The only possible exception would be when $\mathbf{r}-\mathbf{r}'$ would go to zero as well in the limit $\Delta L \to 0$.

From the argument in the previous paragraph it follows that the integral over the *b*-side in Eq. (11) can only have a possible contribution from the current density $\mathbf{i}_b(\mathbf{r}')$ if $\mathbf{r}'$ is in the immediate neighborhood of $\mathbf{r}$. Let $\mathcal{O}'$ be the projection of point $\mathbf{r}$ onto the *b*-side, as in Figure 5, so that the distance between $\mathbf{r}$ and $\mathcal{O}'$ equals $\Delta L$. Then we consider a circle with a small radius $\delta$ around $\mathcal{O}'$. Since the surface integral can only have a contibution from this small circle, we can take $\mathbf{i}_b(\mathbf{r}')$ in Eq. (11) out of the integral as $\mathbf{i}_b(\mathbf{r})$, and compute the remaining integral of $\mathbf{e}_z \cdot \nabla g(\mathbf{r}-\mathbf{r}')$, given by Eq. (12), over the small circle. When we adopt polar coordinates $(\rho,\phi)$ around $\mathcal{O}'$, we have $\mathrm{d}S' = \rho\,\mathrm{d}\rho\,\mathrm{d}\phi$. The integral over $\phi$ yields $2\pi$, and for the integral over $\rho$ we make the substitution $t = \rho^2 + (\Delta L)^2$, so that in Eq. (12) we have $|\mathbf{r}-\mathbf{r}'| = \sqrt{t}$. Then the variable $t$ is small for all $\mathbf{r}'$ on the circle, and we can expand the integrand in a Taylor series around $t$ = 0. The integral can then be evaluated, with result

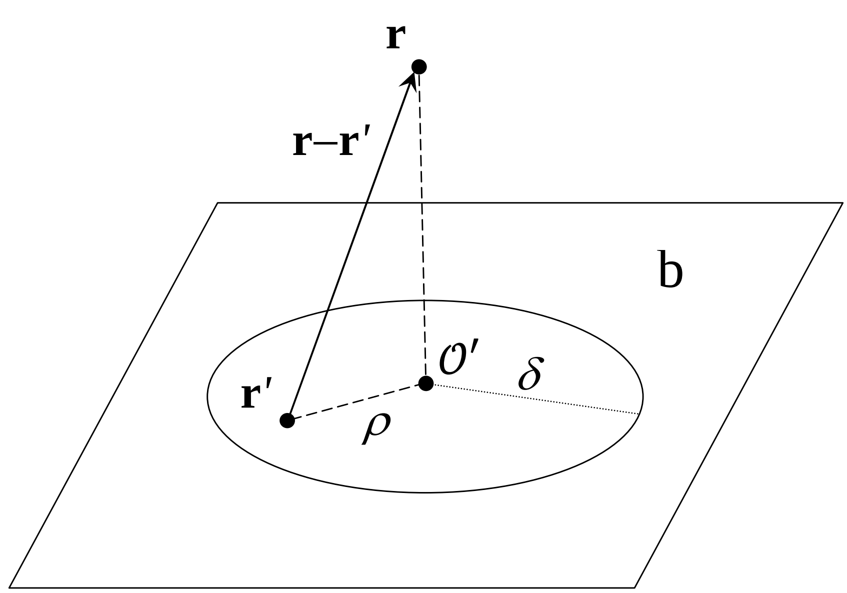


Figure 5. Given point $\mathbf{r}$ at the *a*-side, the integral in Eq. (11) only acquires a contribution from the immediate neighborhood of the projection $\mathcal{O}'$ of $\mathbf{r}$ onto the *b*-side. The integral is evaluated over a circle around $\mathcal{O}'$, using polar coordinates $(\rho,\phi)$ for the point $\mathbf{r}'$. In the limit $\Delta L \to 0$, the integral is independent of the radius $\delta$ of the circle.

$$\frac{1}{2\pi}\int_{\text{circle}} \mathrm{d}S'\,\mathbf{e}_z\cdot\nabla g(\mathbf{r}-\mathbf{r}') = \frac{\Delta L}{\sqrt{\delta^2+(\Delta L)^2}} - 1 + \ldots \; . \tag{13}$$

The ellipses indicate terms that go to zero for $\Delta L \to 0$, $\delta \to 0$. In the limit of a very thin sheet, $\Delta L \to 0$, the first term on the right-hand side of Eq. (13) vanishes for all $\delta$, and when we subsequently let $\delta$ approach zero, the only remaining term on the right-hand side is "-1". The Magnetic Field Integral Equation for a thin sheet thus becomes

$$\mathbf{i}_\mathrm{a}(\mathbf{r}) - \mathbf{i}_\mathrm{b}(\mathbf{r}) = \frac{2}{\mu_\mathrm{o}}\mathbf{e}_z \times \mathbf{B}(\mathbf{r})_\mathrm{inc} \; . \tag{14}$$

This result shows that the current densities at the *a*-side and *b*-side are related, given the incident field. In other words, if the current density at point **r** at the *a*-side is known, so is the current density at the same point **r** at the *b*-side, and vice versa. On the other hand, since the Magnetic Field Integral Equation for a thin sheet reduces to Eq. (14), we find that this equation relates the current densities at the two sides, but it does not yield an equation for either $\mathbf{i}_\mathrm{a}(\mathbf{r})$ or $\mathbf{i}_\mathrm{b}(\mathbf{r})$.

## 5. The Current Densities in a Thin Sheet

The Magnetic Field Integral Equation, Eq. (6), reduces to Eq. (14) when applied to a thin sheet. Since Eq. (14) only relates the current densities at both sides, an additional technique has to be employed to obtain the separate current densities. To this end we note that the current densities generate a magnetic field, given by Eq. (2), where $\mathbf{i}(\mathbf{r})$ has a contribution from both $\mathbf{i}_\mathrm{a}(\mathbf{r})$ and $\mathbf{i}_\mathrm{b}(\mathbf{r})$. However, for $\Delta L \to 0$ the current density at point **r** is effectively the sum

$$\mathbf{i}_\mathrm{s}(\mathbf{r}) = \mathbf{i}_\mathrm{a}(\mathbf{r}) + \mathbf{i}_\mathrm{b}(\mathbf{r}) \; , \tag{15}$$

which is known as the sheet current density [22]. For the generation of the scattered field only $\mathbf{i}_\mathrm{s}(\mathbf{r})$ is relevant, as illustrated in Figure 6. Then in Eq. (2) the current density $\mathbf{i}(\mathbf{r})$ is simply $\mathbf{i}_\mathrm{s}(\mathbf{r})$, and the surface integral runs over the sheet. The requirement that the normal component of the total magnetic field vanishes just outside the medium gives an integral equation for $\mathbf{i}_\mathrm{s}(\mathbf{r})$. Alternatively, we could require that the tangential component of the corresponding electric field is zero, an approach which works particularly well for the Sommerfeld half-plane [23]. Once the sheet current density $\mathbf{i}_\mathrm{s}(\mathbf{r})$ is found, Eq. (15) gives the sum $\mathbf{i}_\mathrm{a} + \mathbf{i}_\mathrm{b}$ and the Magnetic Field Integral Equation (14) gives the difference $\mathbf{i}_\mathrm{a} - \mathbf{i}_\mathrm{b}$. Combining both then yields

$$\mathbf{i}_a(\mathbf{r}) = \tfrac{1}{2}\mathbf{i}_s(\mathbf{r}) + \frac{1}{\mu_o}\mathbf{e}_z \times \mathbf{B}(\mathbf{r})_{inc} \ , \tag{16}$$

$$\mathbf{i}_b(\mathbf{r}) = \tfrac{1}{2}\mathbf{i}_s(\mathbf{r}) - \frac{1}{\mu_o}\mathbf{e}_z \times \mathbf{B}(\mathbf{r})_{inc} \ , \tag{17}$$

in terms of $\mathbf{i}_s(\mathbf{r})$ and the incident magnetic field. Therefore, the current densities at the *a*- and *b*-sides are known as soon as the sheet current density is found. By adding $2\mathbf{i}_b(\mathbf{r})$ to Eq. (14) we can also write

$$\mathbf{i}_s(\mathbf{r}) = \frac{2}{\mu_o}\mathbf{e}_z \times \mathbf{B}(\mathbf{r})_{inc} + 2\mathbf{i}_b(\mathbf{r}) \ . \tag{18}$$

The first term on the right-hand side equals the current density that would be induced in an infinite sheet without openings (mirror, Eq. (10)), and the term $2\mathbf{i}_b(\mathbf{r})$ accounts for the effect of the edges.

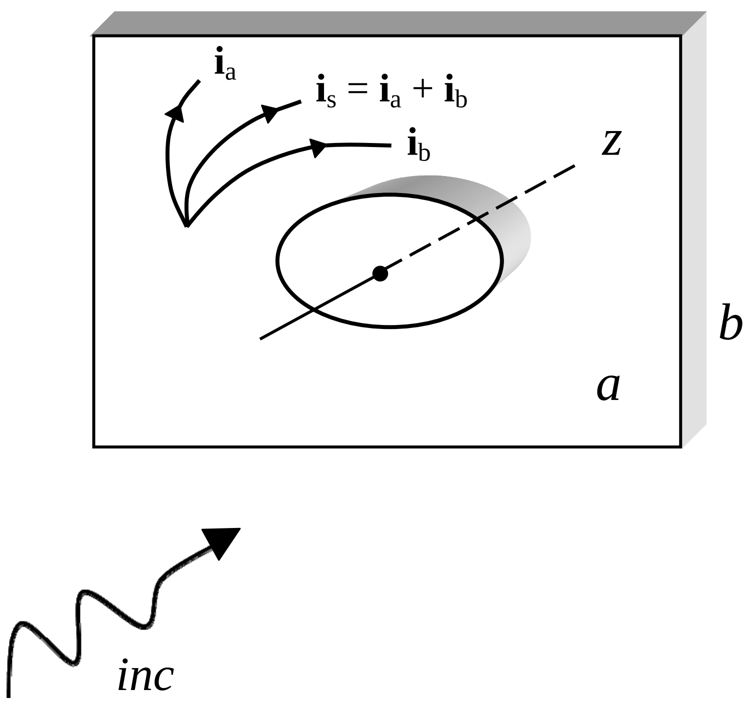


Figure 6. The scattered field is effectively generated by the sheet current density $\mathbf{i}_s$, which is the sum of $\mathbf{i}_a$ and $\mathbf{i}_b$. Equations (15)-(18) show the various relations between these three current densities, indicating that when one of them is known, so are the other two.

## 6. Conclusions

Electromagnetic radiation incident upon a thin sheet induces a current density at both sides. It is shown that for an arbitrary incident field the Magnetic Field Integral Equation relates the current densities at the two sides, and therefore only one of these can be considered as an independent unknown. Alternatively, the total sheet current $\mathbf{i}_S(\mathbf{r})$ determines uniquely the current densities at both sides of the sheet, as shown in Eqs. (16) and (17).

## References

[1] Bohren, C. F. and Huffman, D. R. *Absorption and Scattering of Light by Small Particles,* Wiley, New York, 1983.
[2] Quinten, M., Pack, A. and Wannemacher, R. *Appl. Phys.* B, 68 (1999) 87.
[3] Wang, Z. B., Luk'yanchuk, B. S., Hong, M. H., Lin, Y. and Chong, T. C. *Phys. Rev.* B 70 (2004) 035418.
[4] Stout, B., Nevière, M. and Popov, E. *J. Opt. Soc. Am.* A 22 (2005) 2385.
[5] Stout, B., Nevière, M. and Popov, E. *J. Opt. Soc. Am.* 23 (2006) 1111.
[6] Stout, B., Nevière, M. and Popov, E. *J. Opt. Soc. Am.*. A 23 (2006) 1124.
[7] Sommerfeld, A. *Optics,* Academic Press, New York, 1964, 197.
[8] Jackson, J. D. *Classical Electrodynamics*, 3rd ed., Wiley, New York, 1999, p. 479.
[9] Lalor, É. *Opt. Commun*. 1 (1969) 50.
[10] Sein, J. J. *Opt. Commun.* 2 (1970) 170.
[11] Suzuki, T. *J. Opt. Soc. Am.* 61 (1971) 1029.
[12] Pattanayak, D. N. and Wolf, E. *Opt. Commun.* 6 (1972) 217.
[13] Wolf, E. in Coherence and Quantum Optics, Proc. of the Third Rochester Conference on Coherence and Quantum Optics, L. Mandel and E. Wolf, eds., Plenum, New York, 1973, p. 339.
[14] Sein, J. J. *Opt. Commun*. 14 (1975) 157.
[15] Bowman, J. J., Senior, T. B. A. and Uslenghi, P. L. E. *Electromagnetic and Acoustic Scattering by Simple Shapes*, Hemisphere Publishing, New York, 1987.
[16] Peterson, A. F., Ray, S. L. and Mittra, R. *Computational Methods for Electromagnetics,* IEEE Press, New York, 1998.
[17] Arnoldus, H. F. *J. Opt. Soc. Am.* A 23 (2006) 3063.
[18] Murthy, P. K., Hill, K. C. and Thiele, G. A. *IEEE Trans. Ant. Prop*. 34 (1986) 1173.
[19] Torrungrueng, D., Chou, H-T. and Johnson, J. T. *IEEE Trans. Geo. Remote Sens.* 38 (2000) 1656.
[20] Maue, A. W. and Physik, Z. 126 (1949) 601.
[21] Arnoldus, H. F. *J. Mod. Opt.* 54 (2007) 45.
[22] Lindell, I. V. *Methods for Electromagnetic Field Analysis*, IEEE Press, New York, 1992, p. 77.

[23] Born, M. and Wolf, E. *Principles of Optics*, $6^{th}$ ed., Pergamon, Oxford, 1980, p. 565.