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Reversal of the dipole vortex in a negative index of refraction material

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ABSTRACT

When a small particle is illuminated by a circularly polarized laser beam, the induced electric dipole moment rotates in a plane. The flow lines of the emitted electromagnetic energy are the field lines of the Poynting vector. When the particle is embedded in a dielectric, these field lines have a vortex structure, and the rotation in the vortex has the same orientation as the rotation direction of the dipole. We show that when the embedding medium is a negative index of refraction material, the direction of rotation in the vortex is reversed.

1. Introduction

The electromagnetic response of a medium is accounted for by the (relative) permittivity ε_r and the (relative) permeability μ_r. Both ε_r and μ_r are in general complex, with a non-negative imaginary part, and they depend on the angular frequency ω of the spectral component under consideration. The index of refraction n of the medium is a solution of

$$n^2 = \varepsilon_r \mu_r, \tag{1}$$

and for causality reasons (below) we should take the solution with

$$\mathrm{Im}(n) \geqslant 0. \tag{2}$$

For a dielectric far from resonances, ε_r is approximately real and positive, $\mu_r \approx 1$, and the index of refraction is $n = \sqrt{\varepsilon_r \mu_r}$, which is positive, apart from a very small positive imaginary part. For a metal with ω below the plasmon frequency, the real part of ε_r is negative, the imaginary part of ε_r is small, and $\mu_r \approx 1$. Therefore, n is approximately positive imaginary.

Metamaterials are artificially structured composites, consisting of arrays of sub-wavelength structures, and their electromagnetic response may not be determined only by the material from which they are constructed, but also by the geometry of the design. The typical size of a unit cell of such a composite is well below the wavelength of the radiation under consideration, and this justifies the description of the material as a continuum with permittivity ε_r and permeability μ_r. Of particular interest are metamaterials for which the real parts of both ε_r and μ_r are negative and the imaginary parts of both are small, at a given frequency ω. It then follows from Eqs. (1) and (2) that the real part of the index of refraction n is negative, and such materials are called negative index of refraction materials, or NIM's for short.

In the first experimental demonstration of a negative index of refraction structure [1,2], the composite consisted of split-ring resonators, to obtain a negative μ_r, and a grid of thin metal wires, needed to lower the plasma frequency to the desired range. It was shown that this composite has a negative index of refraction in the microwave range of the electromagnetic spectrum. Many variations in the design structure of the composites have been studied, with attempts to manipulate either the plasma frequency of the permittivity of the metal or the permeability of the split-ring resonators [3–13]. After the successful proof-of-concept demonstrations of the experimental feasibility of constructing a NIM in the microwave region, the quest was on to design composites that operate in the visible region of the spectrum. The design with the split-ring resonators does not scale down to smaller wavelengths, due to increase of loss. New nanostructured materials have been developed, and negative index of refraction has been reported in the THz and near-infrared regions [14–22]. In 2005, the first NIM operating at optical wavelengths was reported [23,24]. The latest designs involve metal-dielectric nanostructures with unit cell widths as small as 10 nm, lattices with coated dielectric spheres, or composites with nanoclusters or nanowires [25–29]. At the present state-of-the-art, loss in the material seems to be the main issue to be addressed in future designs.

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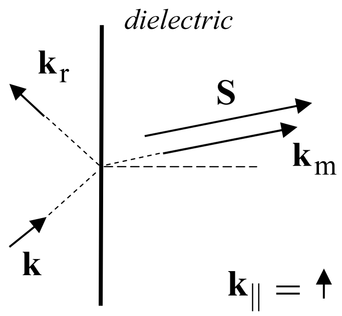

Fig. 1. When a plane wave with wave vector $\mathbf{k}$ is incident upon an interface, it partially reflects and partially transmits into the material. Due to the boundary conditions at the interface, the wave vectors of all waves must have the same parallel component $\mathbf{k}_{\|}$ with respect to the interface. For transmission into a dielectric, this gives the familiar picture shown here.

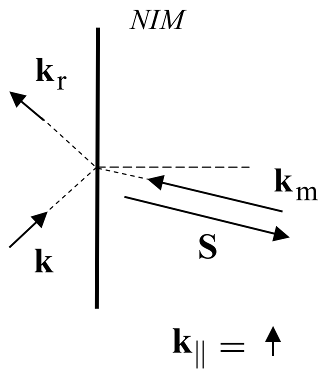

Fig. 2. The same plane wave as in Fig. 1 is now incident upon a material with negative ε_r and μ_r. In a NIM the energy propagates against the wave vector, and therefore the wave vector $\mathbf{k}_m$ in the medium must be as shown here. As a result, the propagation direction of the energy, indicated by the Poynting vector $\mathbf{S}$, of the transmitted wave is at the opposite side of the surface normal, as compared to refraction into a dielectric.

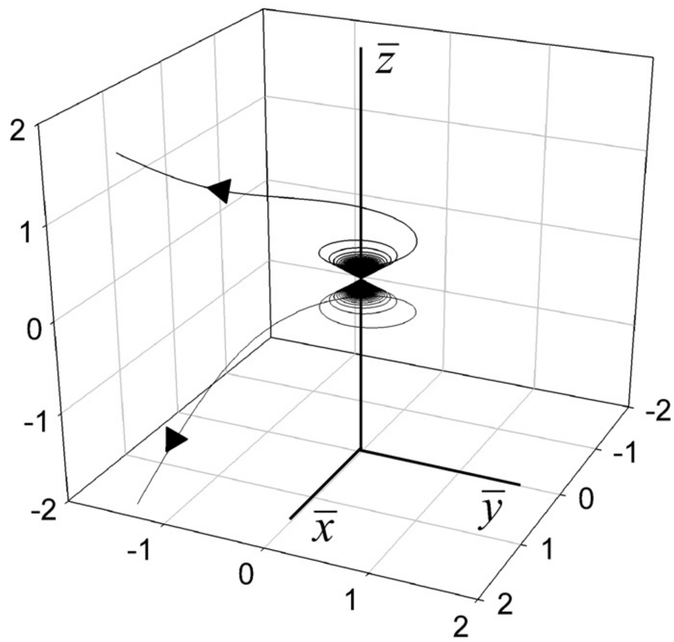

Fig. 3. The figure shows two field lines of the Poynting vector for the radiation emitted by a rotating dipole moment embedded in a dielectric. We use dimensionless variables $\bar{x} = k_o x$, $\bar{y} = k_o y$ and $\bar{z} = k_o z$, so that 2π corresponds to one wavelength. The $\bar{x}$- and $\bar{y}$-axes have been lowered to improve the view. The direction of rotation of the dipole moment is counterclockwise when viewed down the positive z-axis. The field lines swirl around the z-axis while remaining on a cone. The direction of rotation of the field lines is the same as the direction of rotation of the dipole moment.

2. Negative index of refraction

We shall consider time-harmonic fields, oscillating at angular frequency ω. The electric field can then be written as

$$\mathbf{E}(\mathbf{r}, t) = \mathrm{Re}\big[\mathbf{E}(\mathbf{r})e^{-i\omega t}\big], \tag{3}$$

with $\mathbf{E}(\mathbf{r})$ the complex amplitude, and the magnetic field $\mathbf{B}(\mathbf{r}, t)$ can be represented similarly. The time-averaged Poynting vector is defined as

$$\mathbf{S}(\mathbf{r}) = \mathrm{Re}\left[\frac{1}{2\mu}\mathbf{E}(\mathbf{r})^* \times \mathbf{B}(\mathbf{r})\right], \tag{4}$$

with $\mu = \mu_o\mu_r$, and electromagnetic energy flows along the field lines of this vector field. When both ε_r and μ_r are negative, the material is transparent, just like an ordinary dielectric, and Maxwell's equations admit traveling plane-wave solutions. When $\mathbf{k}_m$ is the wave vector of the plane wave, it can be shown easily that in such a material (NIM) the Poynting vector is directed opposite to the wave vector. With the time dependence as in Eq. (3), the phase velocity of the wave is into the direction of the wave vector $\mathbf{k}_m$. Therefore, the phase velocity is opposite to the direction of energy propagation, and such materials are said to have a negative phase velocity. It was shown by Sivukhin [30] that the group velocity has the same direction as the Poynting vector and that materials with negative ε_r and μ_r have opposite group and phase velocities. This property has interesting consequences when an incident plane wave refracts at an interface with a NIM. Fig. 1 shows the refraction for an ordinary dielectric, and Fig. 2 illustrates refraction into a material with negative ε_r and μ_r, e.g., a NIM. The parallel components of all wave vectors have to be the same, due to the boundary conditions. In the NIM, the energy propagates against the wave vector, and since the energy transport has to be away from the interface, the wave vector $\mathbf{k}_m$ of the refracted wave must be as shown in Fig. 2. Consequently, the direction of energy propagation in the negative index of refraction material is into the direction indicated by the Poynting vector $\mathbf{S}$ in the figure. As compared to Fig. 1, the light bends to the other side of the normal to the interface. It was shown for the first time by Mandel'shtam [31] that a negative group velocity leads to negative refraction, and Veselago [32] indicated that this property could be used to construct a lens from a layer of material with negative ε_r and μ_r.

Many other unusual properties have been predicted for NIM's, such as an inverse Doppler shift and Cerenkov effect [33], optical cloacking and the possibility to construct a superlens with a layer of negative index of refraction material [34]. A historical account of negative index of refraction can be found in [35]. In this Letter we wish to add another peculiar property to this list. When a small (compared to a wavelength) particle is embedded in a dielectric and irradiated by a circularly-polarized laser beam, the induced electric dipole moment is a vector which rotates in a plane perpendicular to the propagation direction of the beam (taken to be the z-axis). The field lines of the Poynting vector of the emitted electric dipole radiation are curves which swirl around the z-axis and each field line lies on a cone [36]. The field lines form a vortex pattern, and two typical field lines are shown in Fig. 3. We shall show that when the particle is embedded in a material with negative ε_r and μ_r, the field lines of energy flow of the same rotating dipole moment wind again around the z-axis and each field line lies on a cone, but the direction of rotation around the z-axis is reversed as compared to the field lines for emission in a dielectric. This is shown in Fig. 4.

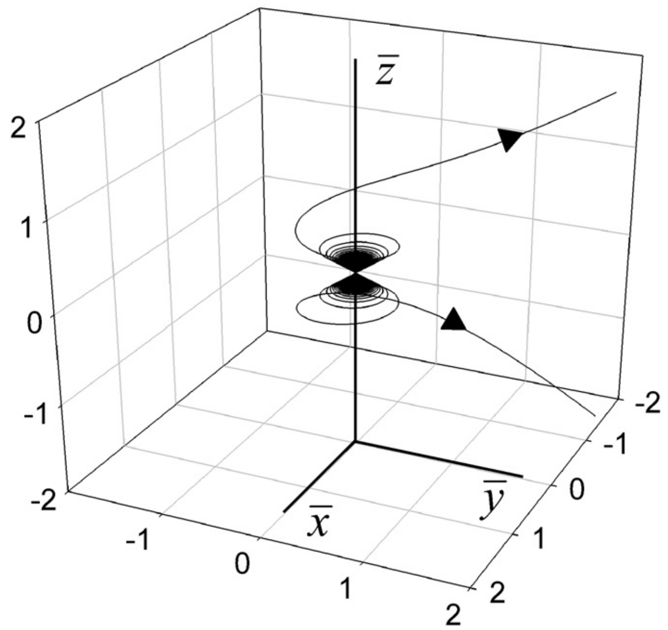

Fig. 4. Shown are energy flow field lines for emission of radiation by the same dipole moment as in Fig. 3, but now the particle is embedded in a material with negative ε_r and μ_r. The direction of rotation of the field lines around the z-axis is reversed, as compared to the rotation of the field lines in Fig. 3.

3. The Green's function and the index of refraction

The electric and magnetic fields of the radiation emitted by a particle embedded in an infinite medium with permittivity ε_r and permeability μ_r are solutions of Maxwell's equations. These solutions can be expressed in terms of the Green's function $g(\mathbf{r})$ for the scalar Helmholtz equation. This function is the solution of

$$\left(\nabla^2 + n^2 k_o^2\right) g(\mathbf{r}) = -4\pi\,\delta(\mathbf{r}), \tag{5}$$

with $k_o = \omega/c$ and n^2 is given by Eq. (1). A solution of Eq. (5) is

$$g(\mathbf{r}) = \frac{e^{ink_o r}}{r}. \tag{6}$$

This solution involves the index of refraction n. However, Eq. (1) only determines n^2, given ε_r and μ_r, and this leaves an ambiguity for the choice of n. In general, ε_r and μ_r are complex, and therefore also n and n^2 are complex. The two solutions of Eq. (1) differ by a minus sign, and are each others reflection in the origin of the complex plane. The Green's function represents a spherical wave, centered at the origin of coordinates, and causality requires that such a wave cannot grow exponentially in amplitude with increasing r. It then follows from (6) that we need to take the solution n for which $\mathrm{Im}(n) \geqslant 0$. A moment of thought then shows that this still leaves an ambiguity for the choice of n when the product $\varepsilon_r\mu_r$ is positive. Causality requires that the imaginary parts of ε_r and μ_r are non-negative and we then see that we can only have $\varepsilon_r\mu_r > 0$ if ε_r and μ_r are both positive or both negative. In order to resolve this ambiguity, we note that both ε_r and μ_r will still have a very small positive imaginary part, representing damping in the material. By taking the limit where these imaginary parts vanish, we find that for $\varepsilon_r\mu_r > 0$ the solution of Eq. (1) should be taken as

$$n = \sqrt{\varepsilon_r\mu_r}, \qquad \varepsilon_r \text{ and } \mu_r \text{ positive} \qquad \text{(dielectric)}, \tag{7}$$

$$n = -\sqrt{\varepsilon_r\mu_r}, \qquad \varepsilon_r \text{ and } \mu_r \text{ negative} \qquad \text{(NIM)}. \tag{8}$$

When a time dependence as in Eq. (3) is considered, the Green's function leads to spherical waves of the form $\exp[i(nk_o r - \omega t)]/r$. For a dielectric we have $n > 0$, and such a wave is an outgoing wave with phase velocity c/n. For $\mathrm{Re}(n) < 0$ this is an incoming wave rather than an outgoing wave and therefore the phase velocity is inward, or negative. This situation is reminiscent of the case for a plane wave, as shown in Fig. 2, where the energy propagates against the wave vector.

4. Electric dipole radiation

The induced electric dipole moment of a particle can be written as

$$\mathbf{d}(t) = \mathrm{Re}\left(\mathbf{d}e^{-i\omega t}\right), \tag{9}$$

with $\mathbf{d}$ the complex amplitude. When this dipole is located at the origin of coordinates, the complex amplitude of the current density is $\mathbf{j}(\mathbf{r}) = -i\omega\mathbf{d}\delta(\mathbf{r})$, and the solution of Maxwell's equations for the radiation field emitted by this dipole, embedded in a medium, can be represented as

$$\mathbf{E}(\mathbf{r}) = \mu_r \frac{k_o^2}{4\pi\varepsilon_o}\left[\mathbf{d}g(\mathbf{r}) + \frac{1}{n^2k_o^2}(\mathbf{d}\cdot\nabla)\nabla g(\mathbf{r})\right], \tag{10}$$

$$\mathbf{B}(\mathbf{r}) = \frac{i\omega\mu}{4\pi}\mathbf{d}\times\nabla g(\mathbf{r}), \tag{11}$$

in terms of the Green's function $g(\mathbf{r})$. With expression (6) for $g(\mathbf{r})$ the derivatives of the Green's function can be worked out, and this yields

$$\mathbf{E}(\mathbf{r}) = \mu_r \frac{k_o^2}{4\pi\varepsilon_o}\left\{\mathbf{d} - (\hat{\mathbf{r}}\cdot\mathbf{d})\hat{\mathbf{r}} + \left[\mathbf{d} - 3(\hat{\mathbf{r}}\cdot\mathbf{d})\hat{\mathbf{r}}\right]\frac{i}{nk_o r}\left(1 + \frac{i}{nk_o r}\right)\right\} g(\mathbf{r}), \tag{12}$$

$$\mathbf{B}(\mathbf{r}) = \frac{n\mu_r}{c}\frac{k_o^2}{4\pi\varepsilon_o}(\hat{\mathbf{r}}\times\mathbf{d})\left(1 + \frac{i}{nk_o r}\right) g(\mathbf{r}). \tag{13}$$

With expressions (12) and (13) for the electric and magnetic field amplitudes, the Poynting vector $\mathbf{S}(\mathbf{r})$ can be constructed. Let us first consider the far field, for which $k_o r \gg 1$. Then only the $O(1/r)$ terms in $\mathbf{E}(\mathbf{r})$ and $\mathbf{B}(\mathbf{r})$ survive, and we obtain

$$\mathbf{S}(\mathbf{r}) \approx \hat{\mathbf{r}}|\mu_r|^2 \frac{ck_o^4}{32\pi^2\varepsilon_o r^2} e^{-2k_o r\,\mathrm{Im}\,n} \times \left[\mathbf{d}^*\cdot\mathbf{d} - (\hat{\mathbf{r}}\cdot\mathbf{d}^*)(\hat{\mathbf{r}}\cdot\mathbf{d})\right]\mathrm{Re}\left(\frac{n}{\mu_r}\right). \tag{14}$$

In the far field, the Poynting vector is proportional to $\hat{\mathbf{r}}$. It can be shown from the discussion above that [37]

$$\mathrm{Re}\left(\frac{n}{\mu_r}\right) \geqslant 0, \tag{15}$$

and since every other term on the right-hand side of Eq. (14) is positive, it follows that the power flow is in the radially outward direction in the far field. For a material with $\mathrm{Re}(n) < 0$ the electric and magnetic fields are spherical incoming waves, but the power flow is in the outward direction, as it should be.

5. The Poynting vector for a dielectric and a NIM

The expression for the Poynting vector for arbitrary ε_r and μ_r is cumbersome, so here we shall only give the result relevant to the present topic. These are the cases shown in Eqs. (7) and (8). For ε_r, μ_r and n positive we have a dielectric and for ε_r, μ_r and n negative we have a NIM. We set $\mathbf{d} = d_o\mathbf{u}$, with $\mathbf{u}\cdot\mathbf{u}^* = 1$, for the dipole moment, and we introduce $q = k_o r$ as the dimensionless distance between the dipole and the field point $\mathbf{r}$. The Poynting vector then becomes

$$\mathbf{S}(\mathbf{r}) = \frac{3P_o}{8\pi r^2}\left\{\mu_r n\left[1-(\hat{\mathbf{r}}\cdot\mathbf{u}^*)(\hat{\mathbf{r}}\cdot\mathbf{u})\right]\hat{\mathbf{r}} + \frac{2\mu_r}{q}\left(1+\frac{1}{n^2q^2}\right)\mathrm{Im}(\hat{\mathbf{r}}\cdot\mathbf{u}^*)\mathbf{u}\right\}, \tag{16}$$

where

$$P_o = \frac{ck_o^4 d_o^2}{12\pi\varepsilon_o}, \tag{17}$$

equals the power emitted by the dipole in free space.

When $\mathbf{u}$ is real we have a linear dipole oscillating back and forth along the vector $\mathbf{u}$, as can be seen from Eq. (9). Then $\mathrm{Im}(\hat{\mathbf{r}}\cdot\mathbf{u}^*)\mathbf{u} = 0$, and $\mathbf{S}(\mathbf{r})$ is in the radial outward direction (since $\mu_r n > 0$ for both cases). The field lines of $\mathbf{S}(\mathbf{r})$ are straight lines coming out of the dipole. We now consider the more interesting case of a rotating dipole moment. When we take

$$\mathbf{u} = -\frac{1}{\sqrt{2}}(\mathbf{e}_x + i\mathbf{e}_y), \tag{18}$$

then it can be verified from Eq. (9) that $\mathbf{d}(t)$ is a vector which rotates counterclockwise in the xy-plane, when viewed down the z-axis. The Poynting vector becomes

$$\mathbf{S}(\mathbf{r}) = \frac{3P_o}{8\pi r^2}\left[\mu_r n\left(1-\tfrac{1}{2}\sin^2\theta\right)\hat{\mathbf{r}} + \left(1+\frac{1}{n^2q^2}\right)\frac{\mu_r\sin\theta}{q}\mathbf{e}_\phi\right], \tag{19}$$

with θ the polar angle with the z-axis, and $\mathbf{e}_\phi$ is the unit vector into the direction of increasing ϕ (angle around the z-axis in the counterclockwise direction). Apart from the radial component, the Poynting vector now has a contribution proportional to $\mathbf{e}_\phi$, and this gives a rotation of the field lines around the z-axis. For a dielectric we have $\mu_r > 0$, and this rotation is in the counterclockwise direction, which is the same orientation as the rotation of the dipole moment. For a negative index of refraction material we have $\mu_r < 0$, and the field lines swirl around the z-axis in the opposite direction as the rotation direction of the dipole moment. Figs. 3 and 4 show two field lines each for this rotating dipole moment.

6. Conclusions

The complex amplitudes of the electric and magnetic fields for the radiation emitted by an electric dipole embedded in a medium with arbitrary values of ε_r and μ_r are given by Eqs. (12) and (13). The Poynting vector can then be obtained by substitution of these expressions into the right-hand side of Eq. (4). For the case where both ε_r and μ_r are positive (dielectric) or where both are negative (NIM), the result is given by Eq. (16). For a linear dipole the field lines of $\mathbf{S}(\mathbf{r})$ are straight lines, coming out of the dipole. When the embedding medium is a material with negative ε_r and μ_r, the spherical waves are incoming, whereas the energy flow is outward. This is very similar to the case of a plane wave, as shown in Fig. 2, where the direction of energy flow is opposite to the wave vector. For a rotating dipole moment embedded in a material with negative ε_r and μ_r, we found that the direction of rotation of the field lines around the z-axis is opposite to the direction of rotation of the dipole moment, whereas for a dielectric both the field lines and the dipole moment have the same orientation in their rotation.

In order to set the dipole moment in rotation, it has to be driven by a circularly polarized laser beam inside the material. It can be shown that when a circularly polarized beam is incident upon the interface between vacuum and a material (dielectric or NIM), the transmitted beam in the material has the same helicity as the incident beam. The induced dipole moment has the same direction of rotation as the direction of rotation of the electric field of the beam. Therefore, the field lines of the Poynting vector have the same orientation in their rotation as the helicity of the driving beam when the medium is a dielectric, but for a NIM the direction of rotation of the field lines of energy flow is opposite to the direction of rotation of the electric field of the incident beam.

References

[1] D.R. Smith, W.J. Padilla, D.C. Vier, S.C. Nemat-Nasser, S. Schultz, Phys. Rev. Lett. 84 (2000) 4184.
[2] R.A. Shelby, D.R. Smith, S. Schultz, Science 292 (2001) 77.
[3] J.B. Pendry, A.J. Holden, W.J. Steward, I. Youngs, Phys. Rev. Lett. 76 (1996) 4773.
[4] J.B. Pendry, A.J. Holden, D.J. Robbins, W.J. Steward, J. Phys.: Condens. Matter 10 (1998) 4785.
[5] J.B. Pendry, A.J. Holden, D.J. Robbins, W.J. Steward, IEEE Trans. Microwave Theory Tech. 47 (1999) 2075.
[6] C.G. Parazolli, R.B. Greegor, K. Li, B.E.C. Koltenbah, M. Tanielian, Phys. Rev. Lett. 90 (2003) 107401.
[7] A.A. Houck, J.B. Brock, I.L. Chang, Phys. Rev. Lett. 90 (2003) 137401.
[8] J.D. Baena, R. Marqués, F. Medina, Phys. Rev. B 69 (2004) 014402.
[9] T.J. Yen, W.J. Padilla, N. Fang, D.C. Vier, D.R. Smith, J.B. Pendry, D.N. Basov, X. Zhang, Science 303 (2004) 1494.
[10] S. Linden, C. Enkrich, M. Wegener, J. Zhou, T. Koschny, C.M. Soukoulis, Science 306 (2004) 1351.
[11] I. Bulu, H. Caglayan, E. Ozbay, Opt. Exp. 13 (2005) 10238.
[12] E. Ekmekci, K. Topalli, T. Akin, G. Turhan-Sayan, Opt. Exp. 17 (2009) 16046.
[13] Y. Yuan, B.-I. Popa, S.A. Cummer, Opt. Exp. 17 (2009) 16135.
[14] S. Zhang, W. Fan, B.K. Minhas, A. Frauenglass, K.J. Malloy, S.R.J. Brueck, Phys. Rev. Lett. 94 (2005) 037402.
[15] G. Dolling, C. Enkrich, M. Wegener, J.F. Zhou, C.M. Soukoulis, S. Linden, Opt. Lett. 30 (2005) 3198.
[16] W.J. Padilla, A.J. Taylor, C. Highstrete, M. Lee, R.D. Averitt, Phys. Rev. Lett. 96 (2006) 107401.
[17] G. Dolling, C. Enkrich, M. Wegener, C.M. Soukoulis, S. Linden, Opt. Lett. 31 (2006) 1800.
[18] M.S. Wheeler, J.S. Aitchison, M. Mojahedi, Phys. Rev. B 73 (2006) 045105.
[19] E. Kim, Y.R. Shen, W. Wu, E. Ponizovskaya, Z. Yu, A.M. Bratkovsky, S.-Y. Wang, R.S. Williams, Appl. Phys. Lett. 91 (2007) 173105.
[20] L. Menon, W.T. Lu, A.L. Friedman, S.P. Bennett, D. Heiman, S. Sridhar, Appl. Phys. Lett. 93 (2008) 123117.
[21] J. Valentine, S. Zhang, T. Zentgraf, E. Ulin-Avila, D.A. Genov, G. Bartal, X. Zhang, Nature 455 (2008) 376.
[22] D.Ö. Güney, T. Koschny, M. Kafesaki, C.M. Soukoulis, Opt. Lett. 34 (2009) 506.
[23] V.M. Shalaev, W. Cai, U.K. Chettiar, H.-K. Yuan, A.K. Sarychev, V.P. Drachev, A.V. Kildishev, Opt. Lett. 30 (2005) 3356.
[24] S. Zhang, W. Fan, N.C. Panoiu, K.J. Malloy, R.M. Osgood, S.R.J. Brueck, Phys. Rev. Lett. 95 (2005) 137404.
[25] C. Enkrich, M. Wegener, S. Linden, S. Burger, L. Zschiedrich, F. Schmidt, J.F. Zhou, Th. Koschny, C.M. Soukoulis, Phys. Rev. Lett. 95 (2005) 203901.
[26] G. Dolling, M. Wegener, C.M. Soukoulis, S. Linden, Opt. Lett. 32 (2007) 53.
[27] U.K. Chettiar, A.V. Kildishev, H.-K. Yuan, W. Cai, S. Xiao, V.P. Drachev, V.M. Shalaev, Opt. Lett. 32 (2007) 1671.
[28] J. Yao, Z. Liu, Y. Liu, Y. Wang, C. Sun, G. Bartal, A.M. Stacy, X. Zhang, Science 321 (2008) 930.
[29] Q. Wu, W. Park, Appl. Phys. Lett. 92 (2008) 153114.
[30] D.V. Sivukhin, Opt. Spectr. 3 (1957) 308.
[31] L.I. Mandel'shtam, Zh. Eksp. Teor. Fiz. 15 (1945) 475.
[32] V.G. Veselago, Sov. Phys. Usp. 10 (1968) 509.
[33] V.E. Pafomov, Sov. Phys. JETP 36 (1959) 1321.
[34] J.B. Pendry, Phys. Rev. Lett. 85 (2000) 3966.
[35] V.M. Agranovich, Y.N. Gartstein, Phys. Usp. 49 (2006) 1029.
[36] H.F. Arnoldus, J.T. Foley, Opt. Commun. 231 (2004) 115.
[37] M.W. McCall, A. Lakhtakia, W.S. Weiglhofer, Eur. J. Phys. 23 (2002) 353.