Optical vortices and singularities due to interference in atomic radiation near a mirror

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We consider radiation emitted by an electric dipole close to a mirror. We have studied the field lines of the Poynting vector, representing the flow lines of the electromagnetic energy, and we show that numerous singularities and subwavelength optical vortices appear in this energy flow pattern. We also show that the field line pattern in the plane of the mirror contains a singular circle across which the field lines change direction.

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A singularity in an optical radiation field is a point where the intensity is zero. A particular interesting phenomenon is the optical vortex, which looks like a whirlpool (2D) or a corkscrew (3D) of light, and such a vortex has a singularity at its center. Optical vortices have been shown to exist in Laguerre–Gaussian laser beams [1], and they have been observed in the interference pattern of incident and reflected waves near a planar interface. Many different ways to generate such interference vortices have been reported [2–6]. In this Letter we shall consider the energy flow pattern of light emitted by an atomic (point) source close to a mirror. The emitted radiation interferes with the reflected light, and as a result singularities and interference vortices appear in the neighborhood of the source and in the region between the source and the surface of the mirror.

The field lines of energy flow are determined by the time-averaged Poynting vector [7], which is defined as

$$\mathbf{S}(\mathbf{r}) = \frac{1}{2\mu_o}\,\mathrm{Re}[\mathbf{E}(\mathbf{r})\times\mathbf{B}(\mathbf{r})^*], \tag{1}$$

with $\mathbf{E}(\mathbf{r})$ and $\mathbf{B}(\mathbf{r})$ the complex electric and magnetic field amplitudes, respectively. The simplest example of atomic radiation is electric dipole radiation, for which the electric and magnetic field amplitudes are well known [8]. The dipole moment $\mathbf{d}_r(t)$ oscillates linearly with angular frequency ω along an axis specified by the unit vector $\boldsymbol{\varepsilon}_r$, so that we have $\mathbf{d}_r(t) = d_o\boldsymbol{\varepsilon}_r\cos(\omega t)$. As illustrated in Fig. 1, we take the surface of the mirror as the xy plane, and the dipole is located on the z axis, a distance h from the mirror. We shall use the inverse wave number, c/ω, as the unit of length, and therefore a dimensionless distance of 2π corresponds to one optical wavelength. The dipole moment oscillates in the xz plane under an angle γ with the z axis, so that we have $\boldsymbol{\varepsilon}_r = \mathbf{e}_x\sin\gamma + \mathbf{e}_z\cos\gamma$. The reflected field in the region $z > 0$ is identical to the field of an image dipole, located a distance h below the mirror, and with an image dipole moment given by $\mathbf{d}_i(t) = d_o\boldsymbol{\varepsilon}_i\cos(\omega t)$. The direction of oscillation of the image is $\boldsymbol{\varepsilon}_i = -\mathbf{e}_x\sin\gamma + \mathbf{e}_z\cos\gamma$ [9]. The magnetic field radiated by the source is

$$\mathbf{B}_r(\mathbf{r}) = -\frac{d_o\omega^3}{4\pi\varepsilon_o c^4}\left(1+\frac{i}{q_r}\right)\frac{e^{iq_r}}{q_r}\boldsymbol{\varepsilon}_r\times\hat{\mathbf{q}}_r, \tag{2}$$

with q_r the distance between the dipole and the field

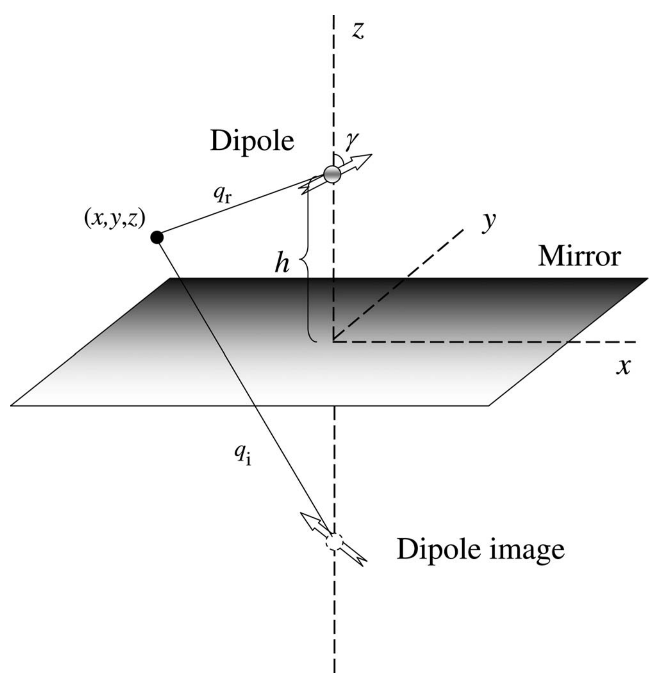

Fig. 1. We use an xyz coordinate system as shown in the diagram. The distance between the source and the observation point is indicated by q_r, and q_i is the distance between the observation point and the mirror dipole.

point, in dimensionless coordinates, and $\hat{\mathbf{q}}_r$ is the unit vector into the direction of the field point and originating at the location of the dipole. The magnetic field of the image is identical in form, with q_r, $\boldsymbol{\varepsilon}_r$, and $\hat{\mathbf{q}}_r$ replaced by q_i, $\boldsymbol{\varepsilon}_i$, and $\hat{\mathbf{q}}_i$. The total magnetic field is then the sum $\mathbf{B}=\mathbf{B}_r+\mathbf{B}_i$. A similar expression can be obtained for the electric field, after which the Poynting vector $\mathbf{S}(\mathbf{r})$ can be constructed.

The Poynting vector $\mathbf{S}(\mathbf{r})$ is a vector field, and the corresponding field lines can be obtained by numerical integration. These field lines are the paths of flow of electromagnetic energy. They will in general be curves in three dimensions, originating from the location of the source, and at a large distance they will asymptotically approach the optical rays in the far field. Because of the reflection symmetry in the xz plane, a field line through a point in the xz plane will remain in the xz plane, and hence the curve becomes a 2D curve. Figure 2 shows the field line pattern in the xz plane for a dipole located at a distance $h=2\pi$ from the mirror and oscillating under an angle of 45° with the z axis. We see from the figure that the flow line pattern contains three optical vortices and a number of singular points. Field lines that leave the dipole in the upward direction are smooth curves, running from the dipole to the far field. The pattern of the field lines leaving the dipole in the downward direction and to the right of the z axis (not shown in the figure) does not exhibit any vortices or other singularities. These field lines bounce off the mirror surface and run to the far field.

Without the mirror, all field lines in Fig. 2 would be straight lines. The curving of the field lines and the appearance of singularities are results of interference between the radiation emitted by the dipole and the radiation reflected at the interface. A particular interesting feature of the flow line pattern is that there are field lines that start at vortex a and end at vortex

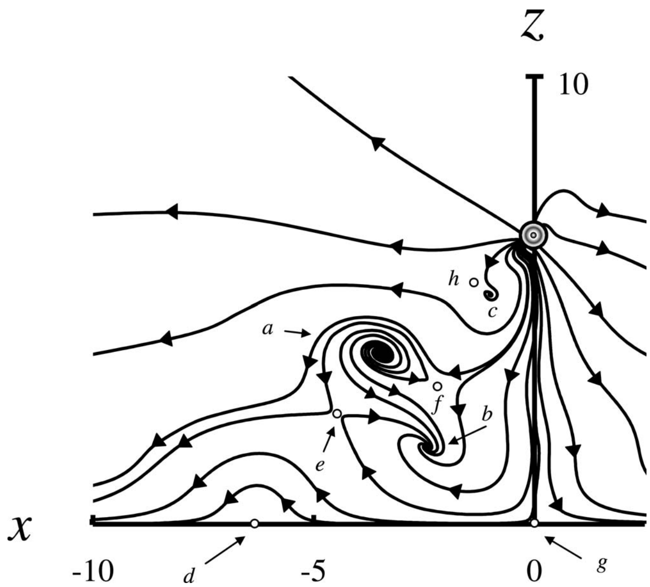

Fig. 2. Field line pattern in the xz plane for radiation emitted by a dipole oscillating under an angle of $\gamma=\pi/4$ with the z axis and located a distance of $h=2\pi$ above the interface.

b; so these field lines do not originate from the location of the dipole, and they do not run to the far field. Furthermore, in the neighborhood of a, all field lines swirl outward, and near b all field lines end at the vortex. This may seem to violate conservation of energy. We have verified numerically that in a 3D graph around the neighborhood of a there are field lines running toward a, providing the inflow of energy at this vortex. Some of the field lines coming out of a run to b, and others run to the far field. This leads to a splitting of the field lines at the singularity labeled e. Similarly, some of the field lines ending at b come from the dipole, and some come from the vortex a. The transition point is again singularity e. At singularity f, the field lines coming from the dipole split. The ones curving upward swing around vortex a and then run to the far field, whereas the field lines that curve down end at vortex b.

For different values of h and γ, the field line pattern is similar, but it may have more or fewer singularities. An exception is the case $\gamma=0$ (dipole oscillating along the z axis), in which case all singularities disappear, except for the singularity directly below the dipole, at the mirror surface (point g in Fig. 2). At a singularity, the Poynting vector vanishes. This can be due to $\mathbf{E}=0$, $\mathbf{B}=0$, $\mathbf{E}\times\mathbf{B}^*=0$, or $\mathbf{E}\times\mathbf{B}^*$ imaginary. For a field point in the xz plane, $\mathbf{B}$ is in the y direction and $\mathbf{E}$ is in the xz plane. The condition $\mathbf{E}=0$ would require that both the x and y components of $\mathbf{E}$ vanish simultaneously, and this would be an unlikely coincidence. For $\mathbf{B}=0$ we have only $B_y=0$, but since B_y is complex this imposes the condition that both the real part and the imaginary parts become zero simultaneously. From Eq. (2), and a similar expression for the image field, we find for $\mathrm{Re}\,B_y=0$ and $\mathrm{Im}\,B_y=0$,

$$\frac{q_r\cos q_r-\sin q_r}{{q_r}^3}(x\cot\gamma-z+h)+\frac{q_i\cos q_i-\sin q_i}{{q_i}^3}(x\cot\gamma+z+h)=0, \quad (3)$$

$$\frac{q_r\sin q_r+\cos q_r}{{q_r}^3}(x\cot\gamma-z+h)+\frac{q_i\sin q_i+\cos q_i}{{q_i}^3}(x\cot\gamma+z+h)=0, \quad (4)$$

respectively. Here, q_r and q_i are defined in Fig. 1. Equations (3) and (4) define two sets of curves in the xz plane, shown in Fig. 3, and at each intersection point we have $\mathbf{B}=0$. Comparing Fig. 2, we see that at the three vortices the Poynting vector is zero because of the vanishing of $\mathbf{B}$. We have verified numerically that at the other singularities the Poynting vector is zero because $\mathbf{E}\times\mathbf{B}^*$ becomes imaginary.

At point d in Fig. 3 we also have $\mathbf{B}=0$, but we see from Fig. 2 that there is no vortex at this point, so this is a singularity of a different nature. It can be

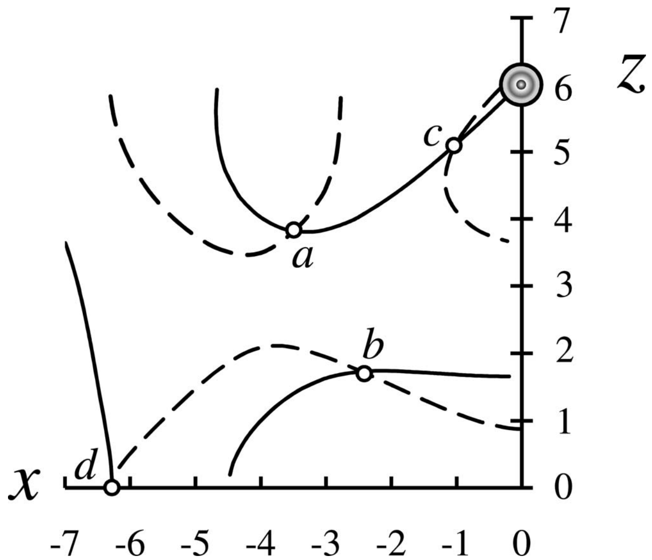

Fig. 3. The solid (dashed) curves represent the solutions of $\text{Re}\,B_y=0$ ($\text{Im}\,B_y=0$) in the xz plane. At the intersections we have $B_y=0$, and therefore $\mathbf{B}=0$ and $\mathbf{S}=0$.

shown from the boundary conditions at a perfect conductor that the Poynting vector at the surface of the mirror is parallel to the mirror, and hence a field line of $\mathbf{S}(\mathbf{r})$ through a point on the mirror surface stays on the surface. Figure 4 shows the field line pattern of the energy flow along the mirror surface for the same parameters as in Fig. 2. It appears that all field lines are straight, and that there is a singular circle in the xy plane across which the field lines change direction. The intersection of this circle with the negative x axis is the singularity d from Figs. 2 and 3. It can be shown that this circle goes through the origin of coordinates, and it follows from Eqs. (3) and (4) that the singularity d on the x axis is located at

$$x=-h\tan\gamma. \tag{5}$$

All field lines in the xy plane are straight lines, and when viewed from a location outside the circle, they all appear to come from the singular point d. However, inside the circle they reverse direction, and the field lines run from a point on the singular circle to the singularity d.

We have shown that interference between dipole radiation and its own reflection at a mirror leads to intricate field line patterns of energy flow, including the appearance of optical vortices with dimensions of the order of the wavelength of the light. When nanoscale resolution of transport of electromagnetic energy is of concern, as for instance in nanophotonics

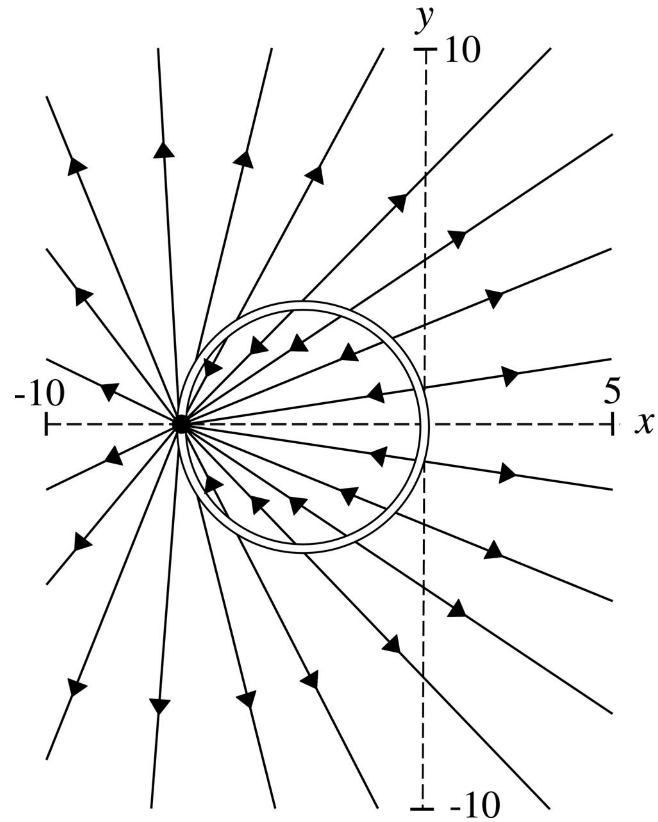

Fig. 4. Energy flow pattern in the xy plane for the same parameters as in Figs. 2 and 3.

device technologies, then it should be taken into consideration that the presence of an interface will dramatically alter the energy flow pattern in the near field, as was illustrated in this Letter for the simple case of a mirror.

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