Far-field detection of the dipole vortex

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The energy flow lines (field lines of the Poynting vector) of electric dipole radiation exhibit a vortex structure in the near field when the dipole moment of the source is in circular rotation. The spatial extend of this vortex is smaller than a wavelength and may not be observable by a measurement in the near field. We show that the rotation of the field lines close to the source affects the image of the dipole in the far field, and this opens the possibility for observation of this vortex by a measurement in the far field.

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Radiation emitted by a localized source appears to travel along straight lines when observed in the far field, and these optical rays are the field lines of the Poynting vector of the electromagnetic field. In the geometrical optics limit of light propagation [1], the optical rays run radially outward from the source as straight lines, no matter the distance to the source. In the exact solution of Maxwell's equations for the radiation from a localized source, the field lines of the Poynting vector only approach straight lines asymptotically, e.g., in the far field. In the vicinity of the source the field lines of the Poynting vector, representing the direction of energy flow, will in general be curves. Each curve will approach a straight line in the far field (many wavelengths from the source), and the corresponding field line runs in the radial direction. In this Letter we shall show by example that the curving of the field lines near the source affects the image of the source in the far field, and this leads to the possibility of observing a near-field property of radiation through detection in the far field. In [2] it was shown that a near-field singularity in the diffraction through a slit may be observed in the far field.

We shall consider an electric dipole at the origin of coordinates, oscillating with angular frequency ω. The time-dependent dipole moment can be written as

$$\mathbf{d}(t) = d_o \,\mathrm{Re}(\boldsymbol{\varepsilon} e^{-i\omega t}) \tag{1}$$

with vector $\boldsymbol{\varepsilon}$ normalized as $\boldsymbol{\varepsilon}\cdot\boldsymbol{\varepsilon}^*=1$ and in general complex. When $\boldsymbol{\varepsilon}$ is real, we have $\mathbf{d}(t)=d_o\boldsymbol{\varepsilon}\cos(\omega t)$, and the dipole moment oscillates along the $\boldsymbol{\varepsilon}$ direction (linear dipole). For a complex-valued $\boldsymbol{\varepsilon}$, the vector $\mathbf{d}(t)$ traces out an ellipse in a plane [3], and this is the most general state of oscillation of a dipole. When $\boldsymbol{\varepsilon}$ is taken as the spherical unit vector

$$\boldsymbol{\varepsilon} = -\frac{1}{\sqrt{2}}(\tau \mathbf{e}_x + i\mathbf{e}_y), \tag{2}$$

the ellipse reduces to a circle, and the parameter τ is the helicity of the rotation (circular dipole). For $\tau=1$ the dipole moment $\mathbf{d}(t)$ rotates counterclockwise in the xy plane when viewed from the positive z axis, and for $\tau=-1$ the rotation is clockwise. When an atom is placed in a circularly polarized laser beam the induced dipole moment of an electronic transition is a circular dipole moment, provided the atomic resonance ω is close to the laser frequency. The emitted resonance fluorescence by such an atom is electric dipole radiation.

With the known expressions for the electric and the magnetic fields for an electric dipole [4], the time-averaged Poynting vector $\mathbf{S}$ at the field point $\mathbf{r}$ can be evaluated. For a linear dipole, $\mathbf{S}$ at point $\mathbf{r}$ is proportional to $\hat{\mathbf{r}}$, the unit vector in the radial direction. Therefore, the field lines of the vector field $\mathbf{S}(\mathbf{r})$ are straight lines, emanating from the location of the dipole. For a circular dipole, the Poynting vector is found to be [5]

$$\mathbf{S}(\mathbf{r}) = \frac{3P_o}{8\pi r^2}\left[\left(1-\frac{1}{2}\sin^2\theta\right)\hat{\mathbf{r}} + \frac{\tau}{q}\left(1+\frac{1}{q^2}\right)\mathbf{e}_\phi \sin\theta\right], \tag{3}$$

where P_o is the emitted power, θ is the angle with the z axis, and $\mathbf{e}_\phi$ is the unit vector in the ϕ direction in a spherical coordinate system. We have set $q=k_o r$, with $k_o=\omega/c$ as the wavenumber, for the dimensionless distance between the dipole and the field point. The term proportional to $\mathbf{e}_\phi$ in Eq. (3) gives rise to a swirling of the field lines around the z axis. A typical field line is shown in Fig. 1, and we see that close to the source, in the near field, the field lines exhibit a vortex structure. We have called this the "dipole vortex" [5]. This vortex structure manifests itself on a subwavelength scale near the location of the dipole.

At a large distance from the dipole, compared to a wavelength, the term proportional to $\mathbf{e}_\phi$ in Eq. (3) vanishes and $\mathbf{S}(\mathbf{r})$ becomes proportional to $\hat{\mathbf{r}}$. Each field line of $\mathbf{S}(\mathbf{r})$ approaches a straight line, which is indicated by l in Fig. 1. However, due to the rotation near the source, the line l does not go through the origin of coordinates but is displaced slightly over a distance comparable to the dimension of the vortex. Therefore, when viewed in the far field, the field line does not appear to come from the site of the dipole, which would be along the direction of line m in Fig. (1), and this will lead to a shift of the image of the

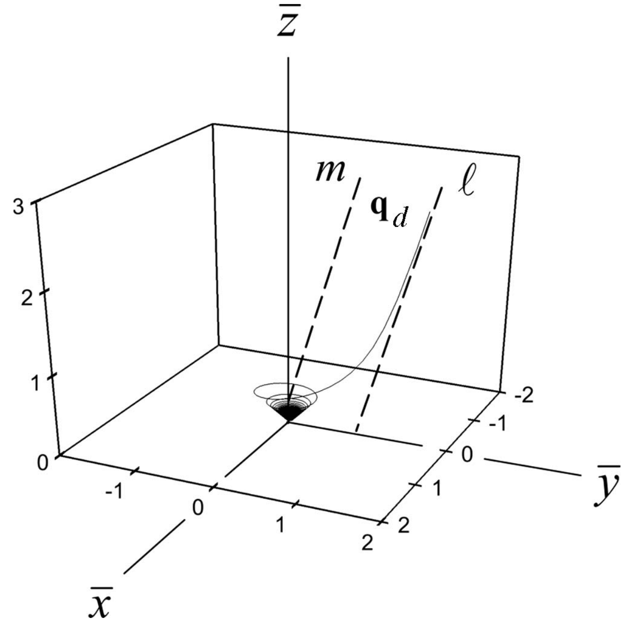

Fig. 1. Shown is the field line of the Poynting vector for $\theta_o = \pi/4$, $\phi_o = \pi$, and $\tau = 1$. The field line lies on the cone $\theta_o = \pi/4$. Near the origin, the field line swirls around the z axis numerous times, and far away from the origin it approaches the line ℓ. The line m is parallel to the line ℓ and intersects the xy plane at the origin. When viewed from the far field, the field line appears to be displaced over vector $\mathbf{q}_d$ as compared to a ray that would come from the location of the dipole. This displacement gives a shift of the image of the dipole.

dipole. To investigate this shift quantitatively, we consider the image that is formed on an observation plane in the far field as shown in Fig. 2. For a given observation direction with spherical coordinates (θ_o, ϕ_o), we take the observation plane perpendicular to the corresponding vector $\hat{\mathbf{r}}_o$, and we set up a rectangular coordinate system with coordinates λ and μ in this plane as shown in the Fig. 2 We shall use dimensionless coordinates $\bar{\lambda} = k_o \lambda$, $\bar{\mu} = k_o \mu$ in this plane.

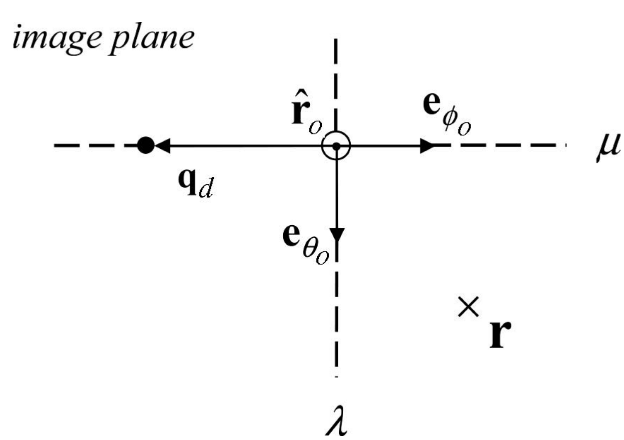

Fig. 2. Image plane in the observation direction (θ_o, ϕ_o) is perpendicular to the radial unit vector $\hat{\mathbf{r}}_o$. The Cartesian coordinates (λ, μ) in this plane are defined as shown. The field line of the Poynting vector that runs into this direction intersects the image plane at the point indicated by the displacement vector $\mathbf{q}_d$, which is along the μ axis (and is in the direction shown for $\tau = 1$).

The field line that runs into the direction (θ_o, ϕ_o) will intersect this plane at a point represented by the displacement vector $\mathbf{q}_d$ in Figs. 1 and 2. Since the rotation of the field lines is around the z axis this vector will be along the μ axis, and such that for positive helicity the displacement is in the negative μ direction. If we set $\mathbf{q}_d = \bar{\mu}_d \mathbf{e}_{\phi_o}$, then it can be shown that [6,7]

$$\bar{\mu}_d = -\frac{2\tau \sin \theta_o}{1 + \cos^2 \theta_o}. \tag{4}$$

The magnitude of the displacement is maximum for observation along the xy plane for which we have $\theta_o = \pi/2$, so $|\bar{\mu}_d| = 2$, and the displacement vanishes for observation along the z axis. In dimensionless coordinates, a distance of 2π corresponds to an optical wavelength, and this shows that the displacement of the field lines in the far field is of a subwavelength order. The displacement is independent of the angle ϕ_o.

The field line of $\mathbf{S}(\mathbf{r})$ that runs asymptotically into the (θ_o, ϕ_o) direction crosses the observation plane at coordinates $(\lambda, \mu) = (0, \mu_d)$, and we may expect that the image of the dipole on this plane is located near this point. Since the image is formed by a bundle of field lines passing through the plane, rather than a single field line, we consider the intensity distribution over the image plane. The intensity at a point $\mathbf{r}$ in the plane, indicated by the point $\mathbf{X}$ in Fig. 2, is given by $I(\mathbf{r}) = \mathbf{S}(\mathbf{r}) \cdot \hat{\mathbf{r}}_o$ since $\hat{\mathbf{r}}_o$ is the unit normal on the plane, and this expression can be evaluated with Eq. (3) for the Poynting vector. The intensity $I(\mathbf{r})$ depends on the angle θ_o between the observation direction and the z axis and on the helicity τ of the dipole but not on the angle ϕ_o. This is due to the rotational symmetry around the z axis. Figure 3 shows the intensity distribution in the $\lambda\mu$ plane for $\tau = 1$ and for an observation point in the xy plane $(\theta_o = \pi/2)$. The $\lambda\mu$ plane is then parallel to the z axis. We find that $I(\mathbf{r})$ has a maximum near the origin in the $\lambda\mu$ plane, and it falls off to zero for λ and μ large. Therefore, this peak represents the brightness of the image spot of the dipole in the $\lambda\mu$ plane. If the field lines would run radially outward from the dipole, as in the ray picture of light propagation, the intensity would have its maximum at the origin of coordinates in the observation plane (for $\theta_o = \pi/2$), but we see from Fig. 3 that the peak is slightly shifted in the negative $\bar{\mu}$ direction. This shift is due to the rotation of the field lines near the site of the dipole as can most easily be understood from Fig. 1. We furthermore notice an asymmetry in the $\bar{\mu}$ direction in Fig. 3, which is also a result of the displacement of the field lines in the μ direction.

The maximum of the intensity distribution in the $\bar{\mu}$ direction appears at the value

$$\bar{\mu}_p = -\frac{2\tau \sin \theta_o}{3 + 5\cos^2 \theta_o}. \tag{5}$$

Therefore, the location of the peak depends on the angle θ_o between the z axis and the observation di-

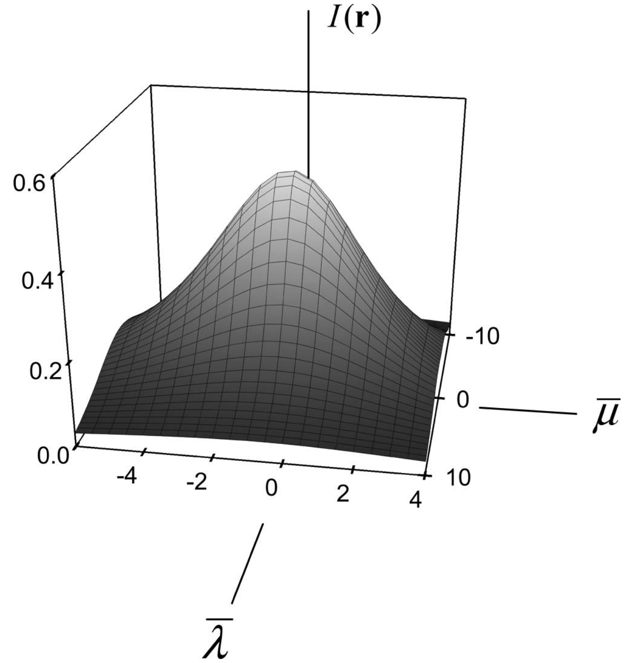

Fig. 3. Shown is the intensity distribution $I(\mathbf{r})$ in the $\bar{\lambda}\bar{\mu}$ plane for $\theta_o=\pi/2$ and $\tau=1$. The peak of the distribution is at the negative side of the $\bar{\mu}$ axis, and this is a result of the spiraling of the field lines in the counterclockwise direction near the position of the dipole.

rection and on the helicity τ of the dipole but not on angle ϕ_o. Equation (5) for the peak position is similar to Eq. (4) for the displacement of the field line in the observation direction (θ_o,ϕ_o). Figure 4 shows both $\bar{\mu}_d$ and $\bar{\mu}_p$ as a function of θ_o, for $\tau=1$. Both the displacement and the shift of the peak are in the same direction along the $\bar{\mu}$ axis, but they are not identical. The maximum shift is $-2/3$, which occurs at $\theta_o=\pi/2$, whereas the displacement at this angle is -2.

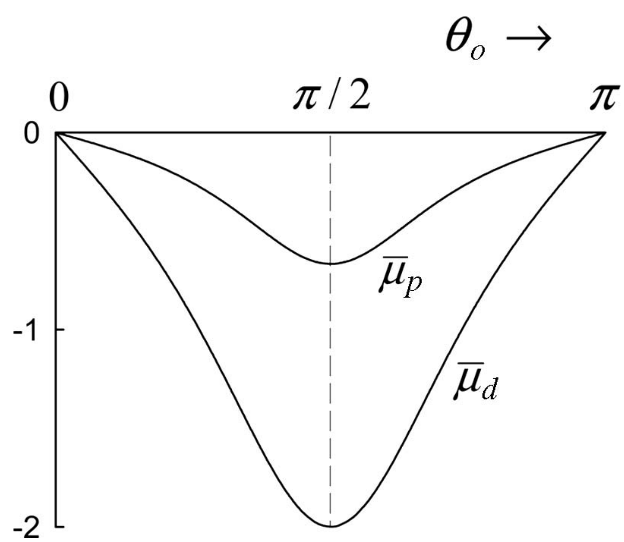

Fig. 4. Shown is the displacement $\bar{\mu}_d$ of the field line in the observation direction (θ_o,ϕ_o) and the location of the peak $\bar{\mu}_p$ of the intensity distribution in the same direction, both as a function of the polar angle of observation θ_o and for $\tau=1$. For $\tau=-1$, both curves change sign but are otherwise the same.

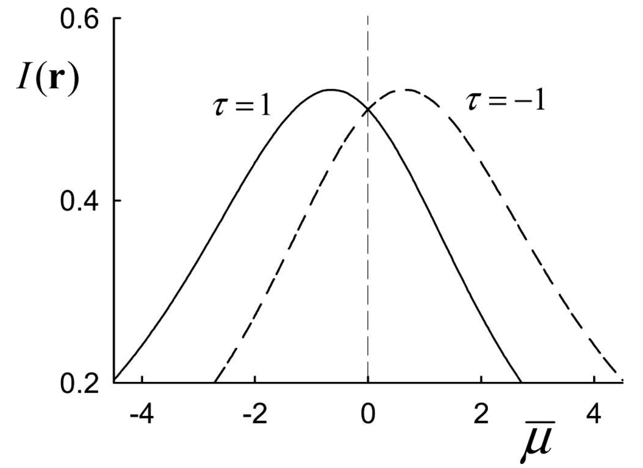

Fig. 5. Graph shows the intensity $I(\mathbf{r})$ as a function of $\bar{\mu}$ for $\bar{\lambda}=0$. The value of the observation angle is $\theta_o=\pi/2$. For $\tau=1$ and $\tau=-1$ the maxima are at $\bar{\mu}_p=-2/3$ and $\bar{\mu}_p=2/3$, respectively.

The shift $\bar{\mu}_p$ of the peak in the far-field image is due to the presence of the vortex in the near field, and this implies the possibility of observing the existence of the dipole vortex by a measurement in the far field. However, a direct measurement of the shift $\bar{\mu}_p$ of the peak may not be feasible since it requires a precise calibration of the location of the origin of coordinates in the observation plane. When an atom radiates dipole radiation in a $\Delta m=-1$ electronic transition, the helicity of the dipole moment is $\tau=1$, whereas for a $\Delta m=1$ transition the helicity is $\tau=-1$. The Δm value of the transition is determined by the polarization of the driving laser. Therefore, if we change the helicity of the laser during an observation, for instance by inserting a half-wave plate in the beam, the value of τ changes sign and so does the location $\bar{\mu}_p$ of the peak. This effect is illustrated in Fig. 5. By changing the helicity of the laser, the peak in the intensity distribution moves over $\pm 2\bar{\mu}_p$, which is $\mp 4/3$ for the case shown in Fig. 5. The observation of such a shift would not require the exact determination of the location of the observation plane. Therefore, such a procedure may allow for the detection of the dipole vortex in the far field.

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