Subwavelength displacement of the far-field image of a radiating dipole

Henk F. Arnoldus,* Xin Li, and Jie Shu

Department of Physics and Astronomy, Mississippi State University, P.O. Drawer 5167, Mississippi State, Mississippi, 39762-5167, USA

**Corresponding author: arnoldus@ra.msstate.edu*

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The field lines of the Poynting vector for light emitted by a dipole with a rotating dipole moment show a vortex pattern near the location of the dipole. In the far field, each field line approaches a straight line, but this line does not appear to come exactly from the location of the dipole. As a result, the image of the dipole in its plane of rotation seems displaced. Secondly, the image in the far field is displaced as compared with the image of a source for which the field lines run radially outward. It turns out that both image displacements are the same. The displacements are of subwavelength scale, and they depend on the angles of observation. The maximum displacement occurs for observation in the plane of rotation and equals λ/π, where λ is the wavelength of the light. © 2008 Optical Society of America

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When a localized source of radiation near the origin of coordinates emits light, a detector in the far field can measure the intensity at observation angles θ_o and ϕ_o in a spherical coordinate system, as shown in Fig. 1. In the far field, the power per unit solid angle is independent of the distance to the source, and it is common to interpret this as a result of the fact that the energy density propagates along a straight line from the source to the observer. This notion is put on firm grounds in the geometrical optics limit for solutions of Maxwell's equations [1]. In this limit, the wavelength λ of the light is assumed to be small, which justifies the neglect of certain terms in Maxwell's equations. The solutions for the electric and magnetic fields then take the form of propagating wave fronts, and the optical rays are defined as the orthogonal trajectories to these wave fronts. In general, these optical rays are curves, but for propagation in a homogeneous medium, like vacuum, the rays are straight lines. On the other hand, the Poynting vector represents the transport of power per unit area. It can then be shown [1] that in the geometrical optics limit the field lines of the Poynting vector coincide with the optical rays, and this supports the picture that energy propagates along straight lines from the source to the detector. Especially when the source is a point source, as in Fig. 1, this conclusion seems obvious.

In this Letter we shall show that radiation emitted by a point source may not propagate along a straight line and that this may lead to an observable effect in the far field. We shall consider an electric dipole, oscillating harmonically with angular frequency ω. The time-dependent dipole moment can be written as

$$\mathbf{d}(t) = d_o \,\mathrm{Re}[\boldsymbol{\varepsilon} e^{-i\omega t}], \tag{1}$$

with $d_o > 0$, and $\boldsymbol{\varepsilon}$ an arbitrary complex-valued constant vector, normalized as $\boldsymbol{\varepsilon}\cdot\boldsymbol{\varepsilon}^* = 1$. The complex amplitudes $\mathbf{E}(\mathbf{r})$ and $\mathbf{B}(\mathbf{r})$ of the electric and magnetic fields of the radiating dipole are well known [2], and the time-averaged Poynting vector

$$\mathbf{S}(\mathbf{r}) = \frac{1}{2\mu_o}\mathrm{Re}[\mathbf{E}(\mathbf{r})\times\mathbf{B}(\mathbf{r})^*] \tag{2}$$

can be evaluated. For an electric dipole at the origin of coordinates we then obtain

$$\mathbf{S}(\mathbf{r}) = \frac{3}{8\pi}\frac{P_o}{r^2}\left\{[1-(\hat{\mathbf{r}}\cdot\boldsymbol{\varepsilon})(\hat{\mathbf{r}}\cdot\boldsymbol{\varepsilon}^*)]\hat{\mathbf{r}} - \frac{2}{q}\left(1+\frac{1}{q^2}\right)\mathrm{Im}[\hat{\mathbf{r}}\cdot\boldsymbol{\varepsilon}]\boldsymbol{\varepsilon}^*\right\}, \tag{3}$$

with P_o being the emitted power, and $\hat{\mathbf{r}}$ being the radially outward unit vector. We have set $q = kr$ for the dimensionless distance between the dipole and the field point $\mathbf{r}$. The wave number is $k = \omega/c = 2\pi/\lambda$, so a dimensionless distance of 2π corresponds to one wavelength. For a magnetic dipole the complex am-

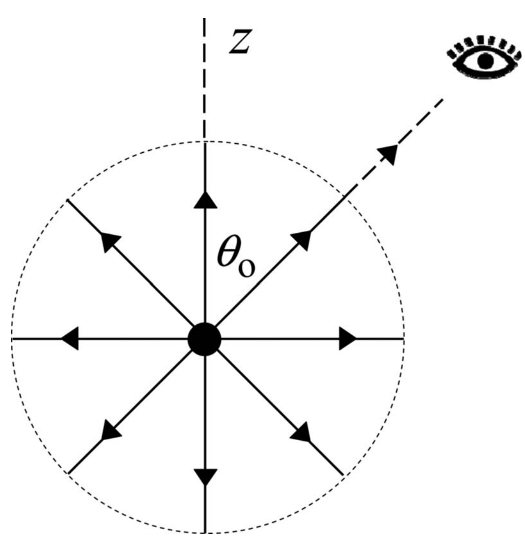

Fig. 1. Source at the origin of coordinates emits light, which is observed in the far field under angles (θ_o, ϕ_o). Angle ϕ_o is not shown in the figure.

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plitudes $\mathbf{E}(\mathbf{r})$ and $\mathbf{B}(\mathbf{r})$ are different, but the resulting expression for the Poynting vector is the same.

When vector $\boldsymbol{\varepsilon}$ is real, the dipole moment is $\mathbf{d}(t) = d_o \boldsymbol{\varepsilon} \cos(\omega t)$. Vector $\mathbf{d}(t)$ oscillates harmonically along the direction of $\boldsymbol{\varepsilon}$, with amplitude d_o. The expression for the Poynting vector simplifies to

$$\mathbf{S}(\mathbf{r}) = \frac{3}{8\pi}\frac{P_o}{r^2}\hat{\mathbf{r}} \sin^2\alpha, \tag{4}$$

with α the angle between $\boldsymbol{\varepsilon}$ and $\hat{\mathbf{r}}$. We see that $\mathbf{S}(\mathbf{r})$ is proportional to $\hat{\mathbf{r}}$, and therefore the field lines are radially outward, as in Fig. 1. Such dipole radiation is emitted in a $\Delta m=0$ electronic transition in an atom. For a $\Delta m = \mp 1$ transition, the vector $\boldsymbol{\varepsilon}$ is a spherical unit vector with respect to the quantization axis [3,4]. When taking this axis as the z axis we have $\boldsymbol{\varepsilon} = -(\tau \mathbf{e}_x + i\mathbf{e}_y)/\sqrt{2}$, with $\tau = \pm 1$. From Eq. (1) it then follows that the dipole moment $\mathbf{d}(t)$ rotates in the xy plane with angular velocity ω, and the rotation is counterclockwise (clockwise) for $\tau=1$ ($\tau=-1$) when viewed from the positive z axis. For such a rotating dipole moment, the Poynting vector becomes [5]

$$\mathbf{S}(\mathbf{r}) = \frac{3}{8\pi}\frac{P_o}{r^2}\left\{\left[1-\frac{1}{2}\sin^2\theta\right]\hat{\mathbf{r}} + \frac{\tau}{q}\left(1+\frac{1}{q^2}\right)\sin\theta\,\mathbf{e}_\phi\right\}. \tag{5}$$

In addition to the radial part, it now has a component proportional to $\mathbf{e}_\phi$, which gives the field lines a rotation about the z axis. Figure 2 shows a typical field line. Close to the dipole the field line pattern exhibits a vortex structure, and in the far field each field line approaches a straight line. The extent of the vortex is well below a wavelength, because a dimensionless distance of 2π corresponds to one optical wavelength λ. Since $\mathbf{S}(\mathbf{r})$ has no component in the $\mathbf{e}_\theta$ direction, a field line lies on a cone $\theta = \theta_o$, with θ_o the angle of observation in the far field.

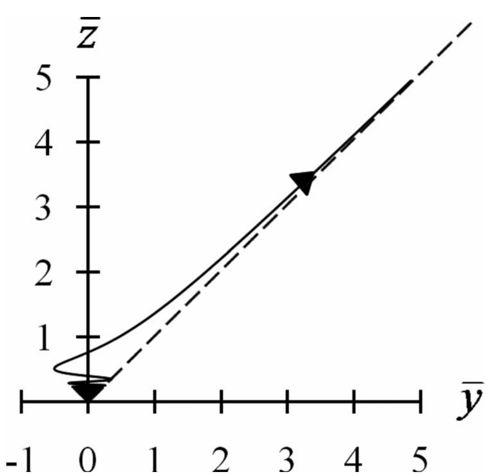

Fig. 2. For a rotating dipole moment at the origin of coordinates, a field line of the Poynting vector has a vortex structure close to the source. Far away from the dipole, the field line approaches a straight line. The dimensionless Cartesian coordinates are defined as $\bar{x}=kx$, $\bar{y}=ky$, and $\bar{z}=kz$.

When viewed from the far field under angles (θ_o, ϕ_o), as shown in Fig. 3, the curved field line of the Poynting vector approaches the straight line l, and for an observer it appears as if the radiation comes from a point in the xy plane, which does not exactly coincide with the location of the source. Let $\mathbf{q}_d$ be the position vector of the image in the xy plane. To find this displacement vector, we need a representation for the line l, which is the asymptote of the field line that is observed under angles (θ_o, ϕ_o). A field line of the vector field $\mathbf{S}(\mathbf{r})$ is the curve through a given point for which $\mathbf{S}(\mathbf{r})$, given by Eq. (5), is on the tangent line at each point along the curve. When we use spherical coordinates (q,θ,ϕ) for an arbitrary point on a field line, then a field line that runs to the observation direction (θ_o, ϕ_o) for q large can be parametrized as

$$\theta = \theta_o, \quad \phi(q) = \phi_o - \tau Z(\theta_o)\frac{1}{q}\left(1+\frac{1}{3q^2}\right), \tag{6}$$

where the dimensionless distance q to the source is the free parameter. Here we have introduced the abbreviation $Z(\theta_o) = 1/(1-\frac{1}{2}\sin^2\theta_o)$, which is a constant along a field line. By considering q large, the field line goes over in its asymptote l, and the intersection between l and the xy plane can be obtained. We find for the displacement vector of the image

$$\mathbf{q}_d = \tau \sin\theta_o Z(\theta_o)(\mathbf{e}_x \sin\phi_o - e_y \cos\phi_o). \tag{7}$$

The magnitude of $\mathbf{q}_d$, which equals $q_d = \sin\theta_o Z(\theta_o)$, depends only on angle θ_o, and is limited by $0 \le q_d \le 2$. For observation along the z axis the displacement vanishes, whereas for observation along the xy plane the displacement is at its maximum of $q_d=2$. The corresponding radial distance to the dipole is λ/π, showing that this virtual displacement of the image is of subwavelength order. Figure 4 shows a field line in the xy plane, and we see that the image is on the y axis when observed from the side of the positive x axis. The direction of $\mathbf{q}_d$ is determined by angle ϕ_o, and by the helicity τ of the dipole. With $\mathbf{q}$ a position vector of a point on l, the equation for the line l

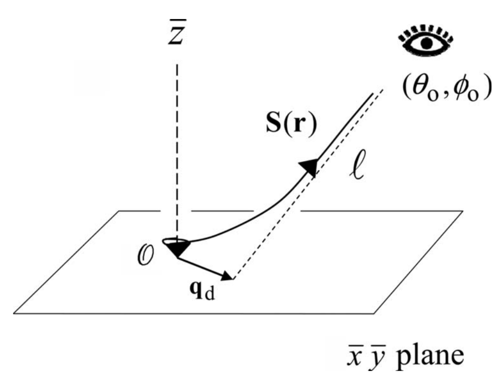

Fig. 3. When a field line of the Poynting vector for a rotating dipole moment is observed in the far field, it appears that the image of the dipole in the xy plane is displaced over vector $\mathbf{q}_d$.

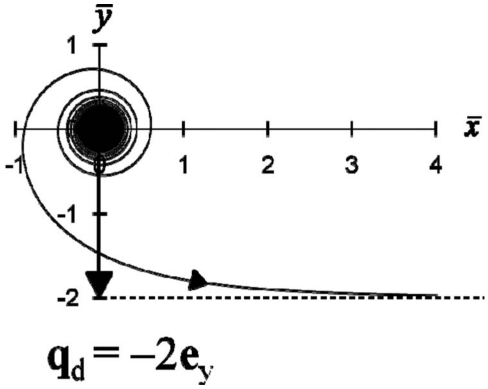

Fig. 4. Field line of the Poynting vector for a rotating dipole moment at the origin of coordinates, which is observed along the x axis. For this case the image in the xy plane has its maximum displacement of λ/π, which corresponds to $q_d = 2$ in dimensionless units.

can be written as $\mathbf{q} = \mathbf{q}_d + \mu \hat{\mathbf{r}}_o$, with μ the free parameter, from which we see that $\mathbf{q}_d$ is perpendicular to the observation direction vector $\hat{\mathbf{r}}_o$.

Let us now consider the displacement of the image in the far field. Figure 5 shows the local coordinate system of an observer, located under angles (θ_o, ϕ_o). If the light would travel along a straight line (ray) from the source to the plane of the observer, the image would be at the origin $\mathcal{O}'$ of the coordinate system. However, owing to the rotation near the source, the image will be displaced, and the field line will intersect the observation plane at a point indicated by the position vector $\mathbf{q}_f$ in this plane. In the far field, this is the same point as the intersection between the plane and line l, and we find this displacement vector to be

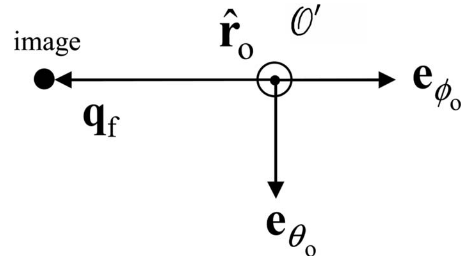

Fig. 5. Image of the dipole in an observation plane in the far field is displaced over vector $\mathbf{q}_f$. It turns out that this displacement vector is the same as $\mathbf{q}_d$ in Fig. 3.

$$\mathbf{q}_f = -\tau \sin \theta_o Z(\theta_o) \mathbf{e}_{\phi_o}. \tag{8}$$

The displacement is in the $-\mathbf{e}_{\phi_o}$ direction for positive helicity and in the $\mathbf{e}_{\phi_o}$ direction for negative helicity. Comparison with Eq. (7) shows that $\mathbf{q}_f = \mathbf{q}_d$, so the displacement in the far field is the same as the displacement of the image in the xy plane. Unlike for a conventional image, the position of the image in the far field depends on the angles of observation (θ_o, ϕ_o).

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