Integral equation formulation for reflection by a mirror

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When light is incident on a mirror, it induces a current density on its surface. This surface current density emits radiation, which is the observed reflected field. We consider a monochromatic incident field with an arbitrary spatial dependence, and we derive an integral equation for the Fourier-transformed surface current density. This equation contains the incident electric field at the surface as an inhomogeneous term. The incident field, emitted by a source current density in front of the mirror, is then represented by an angular spectrum, and this leads to a solution of the integral equation. From this result we derive a relation between the surface current density and the current density of the source. It is shown with examples that this approach provides a simple method for obtaining the surface current density. It is also shown that with the solution of the integral equation, an image source can be constructed for any current source, and as illustration we construct the images of electric and magnetic dipoles and the mirror image of an electric quadrupole. By applying the general solution for the surface current density, we derive an expression for the reflected field as an integral over the source current distribution, and this may serve as an alternative to the method of images.

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1. INTRODUCTION

The reflection of light by objects of all shapes has been the subject of numerous studies [1]. Mie's theory of scattering by a dielectric sphere [2] is still the subject of current research due to its applications in the field of nanoparticles, and the diffraction by a half-plane [3,4] has the attractive feature that its solution can be obtained in closed form (in terms of Fresnel integrals). When the object has a slightly more complex shape, like a wedge [5–7], it becomes increasingly more difficult to analyze the reflected field. A common approach to reflection and diffraction problems is to consider the general solution of Maxwell's equations inside and outside the object with respect to a suitable basis set, and then apply the boundary conditions at the interface. This method, known as the method of moments [8], leads to a set of linear equations for the expansion coefficients, and this set can be solved numerically. In a different approach, integral equations for the electric or magnetic field are derived [9] that have the incident field as an inhomogeneous term. Such equations, which exist in many different forms, can be solved numerically for any given shape of the object. A prime example is the diffraction through a slit, for which the existence of optical vortices in the neighborhood of the slit have been predicted [10,11].

When light is incident on an object, it induces a current density in the material. This current density emits radiation, which is observed as the reflected field outside the object. When the material of the object is a perfect conductor, all current density is at the surface, and this feature allows for a different approach to the problem. For this case, an integral equation for the surface current density can be derived, and it has the incident magnetic field at the surface as an inhomogeneous term. This equation, due to Maue [12], can be solved numerically, and afterward the reflected electric and magnetic fields can be obtained by integration involving the free-space Green's function for the scalar Helmholtz equation. This method has been applied successfully for objects with a large variety of shapes [13–20].

In the examples mentioned above, the incident field is a traveling plane wave, and the object may have a more or less complicated shape. Here we shall consider the complementary problem where the incident field is of arbitrary complexity, but the object is simple. We shall assume a time-harmonic incident electric field of the form

$$\mathbf{E}(\mathbf{r},t) = \mathrm{Re}[\mathbf{E}(\mathbf{r})e^{-i\omega t}], \tag{1}$$

with $\mathbf{E}(\mathbf{r})$ the complex amplitude and ω the angular frequency, and similarly the magnetic field has complex amplitude $\mathbf{B}(\mathbf{r})$. The spatial dependence of $\mathbf{E}(\mathbf{r})$ and $\mathbf{B}(\mathbf{r})$ is restricted only by the requirement that the fields satisfy the free-space Maxwell equations. The field is incident on a mirror of infinite extent, and the material of the mirror is assumed to be a perfect conductor. The incident field induces a surface current density $\mathbf{i}(\mathbf{r},t)$ in the mirror, with complex amplitude $\mathbf{i}(\mathbf{r})$. The surface of the mirror is taken as the xy plane, as shown in Fig. 1, and the z axis is oriented such that the positive side is at the side of the incident field. The surface current density generates the reflected radiation. Of particular interest is the surface current density $\mathbf{i}(\mathbf{r})$ itself. When the source of the incident field is an electric or magnetic dipole, the field lines of $\mathbf{i}(\mathbf{r},t)$ exhibit interesting structures such as singular

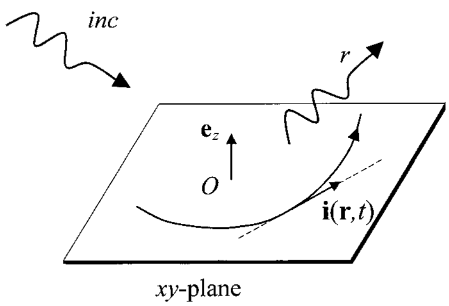

Fig. 1. Electromagnetic field is incident on the surface of a perfect mirror. The surface of the mirror is the xy plane, and the z axis is directed toward the incident field. The incident field induces a current density $\mathbf{i}(\mathbf{r},t)$ on the surface, and this vector field determines a field line pattern on the surface.

points, singular circles, and vortex loops [21,22]. We shall derive an explicit expression for $\mathbf{i}(\mathbf{r})$ in terms of the current distribution of the source, in addition to a similar result for the reflected field.

2. SCATTERED FIELD

The incident field induces a surface current density $\mathbf{i}(\mathbf{r})$ in the mirror, which in turn generates an electromagnetic field. In terms of $\mathbf{i}(\mathbf{r})$, the scattered magnetic field is given by

$$\mathbf{B}(\mathbf{r})_{\mathrm{sc}} = \frac{\mu_o}{4\pi}\nabla \times \int \mathrm{d}S'\mathbf{i}(\mathbf{r}')g(\mathbf{r}-\mathbf{r}'), \tag{2}$$

with $g(\mathbf{r}-\mathbf{r}')$ the free-space Green's function for the Helmholtz equation

$$g(\mathbf{r}-\mathbf{r}') = \frac{e^{ik_o|\mathbf{r}-\mathbf{r}'|}}{|\mathbf{r}-\mathbf{r}'|}, \tag{3}$$

where $k_o = \omega/c$. The corresponding scattered electric field follows from a Maxwell equation as

$$\mathbf{E}(\mathbf{r})_{\mathrm{sc}} = \frac{ic^2}{\omega}\nabla \times \mathbf{B}(\mathbf{r})_{\mathrm{sc}}. \tag{4}$$

In the region $z>0$ this scattered radiation is the reflected field, and in the region $z<0$ it should cancel exactly the incident field.

The representation in Eq. (3) for the Green's function is a spherical wave when considering the $\mathbf{r}$ dependence with fixed source point $\mathbf{r}'$. Alternatively, the Green's function can be written as a superposition of plane waves, which is Weyl's representation [23]. This representation refers to a preferred z direction and is given by

$$g(\mathbf{r}-\mathbf{r}') = \frac{i}{2\pi}\int \mathrm{d}^2\mathbf{k}_\parallel \frac{1}{\beta} e^{i\mathbf{k}_\parallel\cdot(\mathbf{r}-\mathbf{r}')+i\beta|z-z'|}, \tag{5}$$

where the integral runs over the entire $\mathbf{k}_\parallel$ plane, and the parameter β is defined as

$$\beta = \begin{cases} \sqrt{k_o^2-k_\parallel^2}, & k_\parallel < k_o \\ i\sqrt{k_\parallel^2-k_o^2}, & k_\parallel > k_o \end{cases}. \tag{6}$$

A partial wave in Eq. (5) is a traveling wave when β is real. For β imaginary, the partial wave is an evanescent wave that decays exponentially as a function of z, for a given z'. Then we substitute the right-hand side of Eq. (5) into Eq. (2), set $z'=0$, work out the curl, and introduce the wave vectors

$$\mathbf{K}_\pm = \mathbf{k}_\parallel \pm \beta\mathbf{e}_z, \tag{7}$$

which are functions of $\mathbf{k}_\parallel$. This yields for the scattered magnetic field

$$\mathbf{B}(\mathbf{r})_{\mathrm{sc}} = -\frac{\mu_o}{8\pi^2}\int \mathrm{d}^2\mathbf{k}_\parallel \frac{1}{\beta} e^{i\mathbf{K}_\pm\cdot\mathbf{r}}\mathbf{K}_\pm \times \mathbf{I}(\mathbf{k}_\parallel), \tag{8}$$

where the upper (lower) sign is to be used for the region $z>0$ ($z<0$). The transformed current density $\mathbf{I}(\mathbf{k}_\parallel)$ is defined as

$$\mathbf{I}(\mathbf{k}_\parallel) = \int \mathrm{d}S\,\mathbf{i}(\mathbf{r})e^{-i\mathbf{k}_\parallel\cdot\mathbf{r}}, \tag{9}$$

which is the two-dimensional Fourier transform of $\mathbf{i}(\mathbf{r})$. From Eq. (4) we then obtain the scattered electric field

$$\mathbf{E}(\mathbf{r})_{\mathrm{sc}} = \frac{1}{8\pi^2\epsilon_o\omega}\int \mathrm{d}^2\mathbf{k}_\parallel \frac{1}{\beta} e^{i\mathbf{K}_\pm\cdot\mathbf{r}}\mathbf{K}_\pm \times [\mathbf{K}_\pm \times \mathbf{I}(\mathbf{k}_\parallel)], \tag{10}$$

in terms of $\mathbf{I}(\mathbf{k}_\parallel)$.

The function $\mathbf{I}(\mathbf{k}_\parallel)$ is the only unknown in the expressions (8) and (10) for the scattered fields. In addition, Eq. (9) can be inverted as

$$\mathbf{i}(\mathbf{r}) = \frac{1}{4\pi^2}\int \mathrm{d}^2\mathbf{k}_\parallel \mathbf{I}(\mathbf{k}_\parallel)e^{i\mathbf{k}_\parallel\cdot\mathbf{r}}, \tag{11}$$

so the current density can also be obtained from the function $\mathbf{I}(\mathbf{k}_\parallel)$.

3. INTEGRAL EQUATION FOR THE TRANSFORMED SURFACE CURRENT DENSITY

The total electric field is the sum of the incident field and the scattered field, both in front of and behind the mirror. The boundary condition for the electric field at the mirror is that the component of the field parallel with respect to the surface vanishes just in front of the mirror. Therefore we have for a point $\mathbf{r}$ just off the mirror and in $z>0$

$$\mathbf{E}(\mathbf{r})_{\mathrm{sc},\parallel} = -\mathbf{E}(\mathbf{r})_{\mathrm{inc},\parallel}. \tag{12}$$

The parallel part of $\mathbf{E}(\mathbf{r})_{\mathrm{sc}}$ at the surface ($z=0$) can be expressed in terms of $\mathbf{I}(\mathbf{k}_\parallel)$ with Eq. (10). The boundary condition (12) then becomes

$$\frac{1}{8\pi^2\epsilon_o\omega}\int \mathrm{d}^2\mathbf{k}_\parallel \frac{1}{\beta} e^{i\mathbf{k}_\parallel\cdot\mathbf{r}}\{k_o^2\mathbf{I}(\mathbf{k}_\parallel) - \mathbf{k}_\parallel[\mathbf{k}_\parallel\cdot\mathbf{I}(\mathbf{k}_\parallel)]\} = \mathbf{E}(\mathbf{r})_{\mathrm{inc},\parallel}. \tag{13}$$

This is an integral equation for the transformed surface current density $\mathbf{I}(\mathbf{k}_\parallel)$ with the incident field as inhomogeneous term. The value of $\mathbf{r}$, representing a point in the xy plane, is a parameter, and the equation has to hold for all $\mathbf{r}$ in the xy plane.

The second boundary condition for a perfect conductor is that the perpendicular component of the magnetic field vanishes just off the surface. Along similar lines, this gives the relation

$$\frac{\mu_o}{8\pi^2}\int \mathrm{d}^2\mathbf{k}_\parallel \frac{1}{\beta} e^{i\mathbf{k}_\parallel\cdot\mathbf{r}}\mathbf{k}_\parallel \times \mathbf{I}(\mathbf{k}_\parallel) = \mathbf{B}(\mathbf{r})_{\mathrm{inc},\perp}. \tag{14}$$

This relation, however, is dependent. When we apply the operation $\mathbf{e}_y\partial/\partial x - \mathbf{e}_x\partial/\partial y$ on both sides of Eq. (13), and use that the incident electric and magnetic fields are related by Maxwell's equations, we obtain Eq. (14). Therefore, any solution $\mathbf{I}(\mathbf{k}_\parallel)$ of Eq. (13) satisfies Eq. (14) automatically.

4. ANGULAR SPECTRUM OF THE INCIDENT FIELD

The solution $\mathbf{I}(\mathbf{k}_\parallel)$ of Eq. (13) depends on the value of the incident electric field at the surface of the mirror. This incident field is emitted by a source in front of the mirror, as shown schematically in Fig. 2. The source has a (volume) current density $\mathbf{j}(\mathbf{r})$ that emits the magnetic field

$$\mathbf{B}(\mathbf{r})_s = \frac{\mu_o}{4\pi}\nabla \times \int \mathrm{d}V' \mathbf{j}(\mathbf{r}')g(\mathbf{r}-\mathbf{r}'), \tag{15}$$

in all directions. We assume that the extent of the source is limited in the z direction, as in the figure. For the source point $\mathbf{r}'$ in Eq. (15) we then have $z_1<z'<z_2$, and we shall consider only the radiated field outside the region $z_1<z'<z_2$. For the field point $\mathbf{r}$ we then have either $z>z_2$ or $z<z_1$. For the Green's function we use Weyl's representation (5), in which we can then set $|z-z'|=\pm(z-z')$, with the upper (lower) sign holding for $z>z_2$ ($z<z_1$). With the notation of Eq. (7) we can then represent the Green's function as

$$g(\mathbf{r}-\mathbf{r}') = \frac{i}{2\pi}\int \mathrm{d}^2\mathbf{k}_\parallel \frac{1}{\beta} e^{i\mathbf{K}_\pm\cdot(\mathbf{r}-\mathbf{r}')}. \tag{16}$$

Then we substitute the right-hand side of Eq. (16) for $g(\mathbf{r}-\mathbf{r}')$ in Eq. (15), which yields for the magnetic field radiated by the source

$$\mathbf{B}(\mathbf{r})_s = -\frac{\mu_o}{8\pi^2}\int \mathrm{d}^2\mathbf{k}_\parallel \frac{1}{\beta} e^{i\mathbf{K}_\pm\cdot\mathbf{r}}\mathbf{K}_\pm \times \mathbf{J}_\pm(\mathbf{k}_\parallel). \tag{17}$$

Here we have introduced the source functions

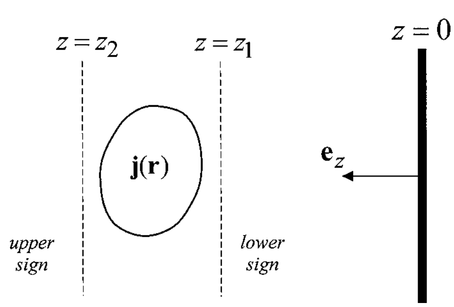

Fig. 2. Current density $\mathbf{j}(\mathbf{r})$ is the source of the incident field, and is confined in the z direction as shown.

$$\mathbf{J}_\pm(\mathbf{k}_\parallel) = \int \mathrm{d}V \mathbf{j}(\mathbf{r})e^{-i\mathbf{K}_\pm\cdot\mathbf{r}} \tag{18}$$

in analogy with Eq. (9) for the transformed surface current density. The expression (17) for the emitted magnetic field involves different source functions for the regions $z>z_2$ and $z<z_1$ (the $\mathbf{J}_+$ and the $\mathbf{J}_-$, respectively), whereas for the surface current sheet we have only a single transformed current density $\mathbf{I}(\mathbf{k}_\parallel)$, which represents the source function for the emission of radiation in both directions away from the mirror. Expressions like Eq. (17) are commonly referred to as angular spectrum representations. The corresponding electric field follows from the magnetic field as in Eq. (4), and we find

$$\mathbf{E}(\mathbf{r})_s = \frac{1}{8\pi^2\epsilon_o\omega}\int \mathrm{d}^2\mathbf{k}_\parallel \frac{1}{\beta} e^{i\mathbf{K}_\pm\cdot\mathbf{r}}\mathbf{K}_\pm \times [\mathbf{K}_\pm \times \mathbf{J}_\pm(\mathbf{k}_\parallel)]. \tag{19}$$

5. SOLUTION OF THE INTEGRAL EQUATION

The mirror is located in the region $z<z_1$, so the incident electric field is given by Eq. (19) with the lower sign. At the mirror we have $z=0$, and when we separate out the parallel part we find

$$\mathbf{E}(\mathbf{r})_{\mathrm{inc},\parallel} = -\frac{1}{8\pi^2\epsilon_o\omega}\int \mathrm{d}^2\mathbf{k}_\parallel \frac{1}{\beta} e^{i\mathbf{k}_\parallel\cdot\mathbf{r}}\{k_o^2\mathbf{J}_-(\mathbf{k}_\parallel)_\parallel - \mathbf{k}_\parallel[\mathbf{K}_-\cdot\mathbf{J}_-(\mathbf{k}_\parallel)]\} \tag{20}$$

for $\mathbf{r}$ in the plane of the mirror. The first term in braces contains the parallel part of $\mathbf{J}_-$ rather than the source function $\mathbf{J}_-$ itself. Then we replace the right-hand side of Eq. (13) by the right-hand side of Eq. (20), and combine the two integrals. We then obtain

$$\int \mathrm{d}^2\mathbf{k}_\parallel \frac{1}{\beta} e^{i\mathbf{k}_\parallel\cdot\mathbf{r}}[k_o^2(\mathbf{I}+\mathbf{J}_{-,\parallel}) - \mathbf{k}_\parallel(\mathbf{k}_\parallel\cdot\mathbf{I}+\mathbf{K}_-\cdot\mathbf{J}_-)] = 0, \tag{21}$$

where we have temporarily suppressed the dependence of the functions on $\mathbf{k}_\parallel$. The left-hand side of Eq. (21) is a two-dimensional Fourier integral, and therefore the function in square brackets is unique. This gives

$$\mathbf{k}_\parallel(\mathbf{k}_\parallel\cdot\mathbf{I}) - k_o^2\mathbf{I} = k_o^2\mathbf{J}_{-,\parallel} - \mathbf{k}_\parallel(\mathbf{K}_-\cdot\mathbf{J}_-), \tag{22}$$

which is now an algebraic equation for $\mathbf{I}(\mathbf{k}_\parallel)$, given the source function $\mathbf{J}_-(\mathbf{k}_\parallel)$.

Equation (22) can be solved for $\mathbf{I}(\mathbf{k}_\parallel)$. To this end we note the vector identity

$$k_\parallel^2\mathbf{I} = \mathbf{k}_\parallel(\mathbf{k}_\parallel\cdot\mathbf{I}) - \mathbf{k}_\parallel\times(\mathbf{k}_\parallel\times\mathbf{I}), \tag{23}$$

which expresses that $\mathbf{I}$ is known as soon as we know its dot and cross products with $\mathbf{k}_\parallel$. By taking $\mathbf{k}_\parallel\cdot(\ldots)$ and $\mathbf{k}_\parallel\times(\ldots)$ of Eq. (22) and using $\beta^2=k_o^2-k_\parallel^2$, we find

$$\mathbf{I}(\mathbf{k}_\parallel) = -\mathbf{J}_-(\mathbf{k}_\parallel)_\parallel - \frac{1}{\beta}[\mathbf{e}_z \cdot \mathbf{J}_-(\mathbf{k}_\parallel)]\mathbf{k}_\parallel \tag{24}$$

as the solution. With some further manipulation this can be written as

$$\mathbf{I}(\mathbf{k}_\parallel) = -\mathbf{J}_-(\mathbf{k}_\parallel) - \frac{1}{\beta}[\mathbf{e}_z \cdot \mathbf{J}_-(\mathbf{k}_\parallel)]\mathbf{K}_-, \tag{25}$$

and upon combining both terms we obtain

$$\mathbf{I}(\mathbf{k}_\parallel) = -\frac{1}{\beta}\mathbf{e}_z \times [\mathbf{K}_- \times \mathbf{J}_-(\mathbf{k}_\parallel)]. \tag{26}$$

This result shows that the transformed surface current density in the mirror can be expressed in a very simple way in terms of the source function $\mathbf{J}_-(\mathbf{k}_\parallel)$.

6. CURRENT DENSITY IN THE MIRROR

With Eqs. (11) and (26) we find for the surface current density

$$\mathbf{i}(\mathbf{r}) = -\frac{1}{4\pi^2}\mathbf{e}_z \times \int \mathrm{d}^2\mathbf{k}_\parallel \frac{1}{\beta} e^{i\mathbf{k}_\parallel\cdot\mathbf{r}}[\mathbf{K}_- \times \mathbf{J}_-(\mathbf{k}_\parallel)], \tag{27}$$

which is an angular spectrum representation for $\mathbf{i}(\mathbf{r})$. Then we substitute the right-hand side of Eq. (18) with the lower sign for $\mathbf{J}_-(\mathbf{k}_\parallel)$, and note the identity

$$\int \mathrm{d}^2\mathbf{k}_\parallel \frac{1}{\beta}\mathbf{K}_- e^{i\mathbf{K}_-\cdot(\mathbf{r}-\mathbf{r}')} = 2\pi\nabla' g(\mathbf{r}-\mathbf{r}'), \quad z' > z, \tag{28}$$

which follows from Eq. (16). This gives

$$\mathbf{i}(\mathbf{r}) = \frac{1}{2\pi}\mathbf{e}_z \times \int \mathrm{d}V' \mathbf{j}(\mathbf{r}') \times [\nabla' g(\mathbf{r}-\mathbf{r}')]. \tag{29}$$

By introducing the function

$$f(\mathbf{r}) = \frac{1}{r^2}\left(ik_o - \frac{1}{r}\right)e^{ik_o r}, \tag{30}$$

which depends only on vector $\mathbf{r}$ though its magnitude r, Eq. (29) simplifies to

$$\mathbf{i}(\mathbf{r}) = \frac{1}{2\pi}\mathbf{e}_z \times \int \mathrm{d}V' f(\mathbf{r}-\mathbf{r}')(\mathbf{r}-\mathbf{r}') \times \mathbf{j}(\mathbf{r}'). \tag{31}$$

Result (31) shows that the surface current density $\mathbf{i}(\mathbf{r})$ for any given $\mathbf{r}$ can be found as an integral over the source current density $\mathbf{j}(\mathbf{r})$. The surface current density $\mathbf{i}(\mathbf{r})$ is induced by the magnetic field that is emitted by the source current density $\mathbf{j}(\mathbf{r})$. With Eq. (31), the surface current density in the mirror can be evaluated without considering the emitted magnetic field first, which is a great simplification.

7. CURRENT DENSITY INDUCED BY MULTIPOLES

A prime example of a localized source is an electric or a magnetic multipole. On one hand, the current distribution of any localized source can be written as a superposition of pointlike multipoles [24], and on the other hand, the radiation emitted by atoms and molecules is multipole radiation. An electric dipole located at $\mathbf{r}_o$ in front of the mirror has a current density

$$\mathbf{j}(\mathbf{r}) = -i\omega\mathbf{d}\,\delta(\mathbf{r}-\mathbf{r}_o), \tag{32}$$

where the dipole moment $\mathbf{d}$ is an arbitrary complex constant vector. The current density it induces in the mirror follows immediately from Eq. (31), and we obtain

$$\mathbf{i}(\mathbf{r}) = -\frac{i\omega}{2\pi}f(\mathbf{r}-\mathbf{r}_o)\mathbf{e}_z \times [(\mathbf{r}-\mathbf{r}_o) \times \mathbf{d}]. \tag{33}$$

This result can also be derived with the method of images. An image dipole behind the mirror produces the reflected field, and then from the boundary conditions at the interface for the magnetic field, the surface current density can be found. The field lines of the corresponding $\mathbf{i}(\mathbf{r},t) = \mathrm{Re}[\mathbf{i}(\mathbf{r})\exp(-i\omega t)]$ depend on the orientation of $\mathbf{d}$ with respect to the surface. These field line patterns were studied in [21], where it was shown that some intriguing spiraling field line structures can appear.

A magnetic dipole with dipole moment $\mathbf{p}$, located in $\mathbf{r}_o$, has a current density

$$\mathbf{j}(\mathbf{r}) = -\mathbf{p} \times \nabla\delta(\mathbf{r}-\mathbf{r}_o). \tag{34}$$

We substitute this in the right-hand side of Eq. (31). The operator ∇ can be eliminated by integration by parts, and this introduces the gradient of the function $f(\mathbf{r}-\mathbf{r}_o)$. When we write

$$\nabla f(\mathbf{r}-\mathbf{r}_o) = (\mathbf{r}-\mathbf{r}_o)h(\mathbf{r}-\mathbf{r}_o), \tag{35}$$

then the function $h(\mathbf{r})$ is explicitly

$$h(\mathbf{r}) = -\frac{1}{r^3}\left(k_o^2 + \frac{3ik_o}{r} - \frac{3}{r^2}\right)e^{ik_o r}. \tag{36}$$

The surface current density then becomes

$$\mathbf{i}(\mathbf{r}) = -\frac{1}{\pi}f(\mathbf{r}-\mathbf{r}_o)\mathbf{e}_z \times \mathbf{p} + \frac{1}{2\pi}h(\mathbf{r}-\mathbf{r}_o)\mathbf{e}_z \times\{(\mathbf{r}-\mathbf{r}_o) \times [(\mathbf{r}-\mathbf{r}_o) \times \mathbf{p}]\}, \tag{37}$$

and with some vector manipulations this can be simplified to

$$\mathbf{i}(\mathbf{r}) = \frac{1}{2\pi}k(\mathbf{r}-\mathbf{r}_o)\mathbf{e}_z \times \mathbf{p} + \frac{1}{2\pi}h(\mathbf{r}-\mathbf{r}_o) \times [(\mathbf{r}-\mathbf{r}_o)\cdot\mathbf{p}]\mathbf{e}_z \times (\mathbf{r}-\mathbf{r}_o), \tag{38}$$

where we have set

$$k(\mathbf{r}) = \frac{1}{r}\left(k_o^2 + \frac{ik_o}{r} - \frac{1}{r^2}\right)e^{ik_o r}, \tag{39}$$

which is $k(\mathbf{r}) = -2f(\mathbf{r}) - r^2 h(\mathbf{r})$. The field line pattern of the corresponding $\mathbf{i}(\mathbf{r},t)$ can be very complicated, and for certain values of $\mathbf{p}$ vortices appear in the surface current [22].

An electric quadrupole at $\mathbf{r}_o$ has a current density

$$\mathbf{j}(\mathbf{r}) = \frac{i\omega}{6}\overleftrightarrow{Q}\cdot\nabla\delta(\mathbf{r}-\mathbf{r}_o), \tag{40}$$

where $\overleftrightarrow{Q}$ is a complex-valued, symmetric Cartesian tensor of rank two. The induced surface current density can be evaluated from Eq. (31), and this yields

$$\mathbf{i}(\mathbf{r}) = \frac{i\omega}{12\pi}h(\mathbf{r}-\mathbf{r}_o)\mathbf{e}_z\times\{(\mathbf{r}-\mathbf{r}_o)\times[(\mathbf{r}-\mathbf{r}_o)\cdot\overleftrightarrow{Q}]\}. \tag{41}$$

Here we have used the identity

$$\sum_{\alpha,\beta} Q_{\alpha\beta}\mathbf{e}_\alpha\times\mathbf{e}_\beta = 0, \quad \alpha,\beta = x,y,z, \tag{42}$$

which follows from the fact that $\overleftrightarrow{Q}$ is symmetric.

As illustration, let us consider an electric quadrupole with a quadrupole moment represented by the matrix

$$\overleftrightarrow{Q} \doteq \frac{Q_o}{\sqrt{6}}\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 2 \end{bmatrix}, \quad Q_o > 0, \tag{43}$$

on the Cartesian basis. This is the quadrupole moment of a $\Delta m=0$ electronic transition in an atom. Let the quadrupole be located on the z axis at a distance H from the mirror so that $\mathbf{r}_o = H\mathbf{e}_z$. The general expression (41) can then be simplified to

$$\mathbf{i}(\mathbf{r}) = -Q_o\frac{i\omega H}{4\pi\sqrt{6}}\mathbf{r}h(\mathbf{r}-\mathbf{r}_o), \tag{44}$$

and the time-dependent current density becomes

$$\mathbf{i}(\mathbf{r},t) = -Q_o\frac{ck_o^6 H}{4\pi\sqrt{6}}\frac{1}{q_1^3}\times\left[\frac{3}{q_1}\cos(q_1-\omega t) + \left(1-\frac{3}{q_1^2}\right)\sin(q_1-\omega t)\right]\mathbf{r}, \tag{45}$$

where we have set

$$q_1 = k_o|\mathbf{r}-\mathbf{r}_o| \tag{46}$$

for the dimensionless distance between the quadrupole at $\mathbf{r}_o$ and the point $\mathbf{r}$ in the xy plane. The current density $\mathbf{i}(\mathbf{r},t)$ is proportional to $\mathbf{r}$, so it is in the radial direction. Therefore, the field lines are radially inward or outward, depending on the sign of the expression in square brackets. This expression is zero when

$$\tan(q_1-\omega t) = \frac{3q_1}{3-q_1^2}, \tag{47}$$

which is a transcendental equation for q_1, given t. The solutions of this equation correspond to circles around the origin, and the field lines of $\mathbf{i}(\mathbf{r},t)$ therefore change direction when crossing such a circle. Since the current density is zero on each circle, field lines start and end on a circle and the circles are singular circles of the field line pattern. Figure 3 shows the field lines of $\mathbf{i}(\mathbf{r},t)$ for a fixed t. When time progresses, the circles expand (at near the speed of light), and the field line picture changes accordingly. New singular circles emanate from the origin of coordinates, which itself is a singular point [$\mathbf{i}(0,t)=0$], and they expand rapidly with time.

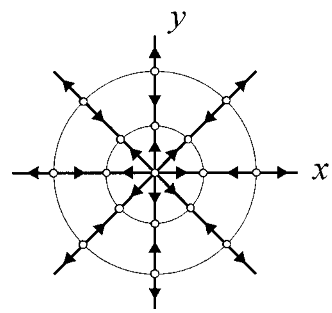

Fig. 3. Field lines of the current density $\mathbf{i}(\mathbf{r},t)$ in the mirror for the case where the source of the incident radiation is an electric quadrupole with a quadrupole tensor given by Eq. (43), and for a fixed time t. The field lines are in the radial direction, either inward or outward, and they change direction across the singular circles, indicated by thin lines.

8. SCATTERED FIELD AND IMAGE SOURCE

The magnetic and electric fields radiated by the surface current density $\mathbf{i}(\mathbf{r})$ are given by Eqs. (8) and (10), respectively, in terms of the transformed surface current density $\mathbf{I}(\mathbf{k}_\parallel)$. For the scattered magnetic field behind the mirror we use Eq. (8) with the lower sign, and when we use the representation (25) for the solution $\mathbf{I}(\mathbf{k}_\parallel)$, we see that $\mathbf{K}_-\times\mathbf{I} = -\mathbf{K}_-\times\mathbf{J}_-$. We then obtain

$$\mathbf{B}(\mathbf{r})_{sc} = \frac{\mu_o}{8\pi^2}\int d^2\mathbf{k}_\parallel\frac{1}{\beta}e^{i\mathbf{K}_-\cdot\mathbf{r}}\mathbf{K}_-\times\mathbf{J}_-(\mathbf{k}_\parallel), \quad z<0. \tag{48}$$

The magnetic field emitted by the source is for the region behind the mirror given by Eq. (17) with the lower sign, and we notice that this field is the opposite of the field in Eq. (48). Therefore, the field by $\mathbf{i}(\mathbf{r})$ cancels exactly the field by $\mathbf{j}(\mathbf{r})$ in the region behind the mirror, as it should.

The field reflected by the mirror is the scattered field in the region $z>0$. The reflected magnetic field is given by Eq. (8) with the upper sign, and with the solution for $\mathbf{I}(\mathbf{k}_\parallel)$ in the form of Eq. (26) we find for the reflected magnetic field

$$\mathbf{B}(\mathbf{r})_r = \frac{\mu_o}{8\pi^2}\int d^2\mathbf{k}_\parallel\frac{1}{\beta^2}e^{i\mathbf{K}_+\cdot\mathbf{r}}\mathbf{K}_+\times\{\mathbf{e}_z\times[\mathbf{K}_-\times\mathbf{J}_-(\mathbf{k}_\parallel)]\}, \tag{49}$$

and a similar expression for the reflected electric field follows from Eq. (10). This reflected field is identical to the field emitted by an image source behind the mirror, as can be shown as follows.

The magnetic field radiated by the source is given by Eq. (17) in terms of the source functions $\mathbf{J}_\pm(\mathbf{k}_\parallel)$. If an image source were located behind the mirror, then the emitted magnetic field would be identical in form to Eq. (17), but with $\mathbf{J}_\pm(\mathbf{k}_\parallel)$ replaced by the corresponding source functions for the image $\mathbf{J}_\pm(\mathbf{k}_\parallel)^{im}$. Since the reflected field travels in the positive z direction, we would need the form of Eq. (17) with the upper sign, and hence the reflected magnetic field would be of the form

$$\mathbf{B}(\mathbf{r})_{\mathrm{r}} = -\frac{\mu_{\mathrm{o}}}{8\pi^2}\int \mathrm{d}^2\mathbf{k}_{\|}\frac{1}{\beta}e^{i\mathbf{K}_+\cdot\mathbf{r}}\mathbf{K}_+ \times \mathbf{J}_+(\mathbf{k}_{\|})^{\mathrm{im}}. \tag{50}$$

Now let us define the function

$$\mathbf{J}_+(\mathbf{k}_{\|})^{\mathrm{im}} = \mathbf{J}_-(\mathbf{k}_{\|})_{\perp} - \mathbf{J}_-(\mathbf{k}_{\|})_{\|}. \tag{51}$$

This function is determined by the –source function of the source itself, and it follows from a reversal of the sign of the parallel component with respect to the xy plane. It can then be shown by inspection that

$$\frac{1}{\beta}\mathbf{K}_+ \times \{\mathbf{e}_z \times [\mathbf{K}_- \times \mathbf{J}_-(\mathbf{k}_{\|})]\} = -\mathbf{K}_+ \times \mathbf{J}_+(\mathbf{k}_{\|})^{\mathrm{im}}, \tag{52}$$

and therefore the magnetic fields in Eqs. (49) and (50) are identical. We conclude that the function $\mathbf{J}_+(\mathbf{k}_{\|})^{\mathrm{im}}$ is the +source function for the image source. We shall see in the next sections that this source is indeed located behind the mirror at the mirror image location.

9. IMAGES OF MULTIPOLES

For an electric dipole $\mathbf{d}$ at position $\mathbf{r}_{\mathrm{o}}$, the current density is given by Eq. (32), and the source functions follow from Eq. (18). This gives

$$\mathbf{J}_{\pm}(\mathbf{k}_{\|}) = -i\omega\mathbf{d}e^{-i\mathbf{K}_{\pm}\cdot\mathbf{r}_{\mathrm{o}}}, \tag{53}$$

and with Eq. (17) we then obtain the angular spectrum of the emitted magnetic field by the dipole:

$$\mathbf{B}(\mathbf{r})_{\mathrm{s}} = \frac{i\omega\mu_{\mathrm{o}}}{8\pi^2}\int \mathrm{d}^2\mathbf{k}_{\|}\frac{1}{\beta}e^{i\mathbf{K}_{\pm}\cdot(\mathbf{r}-\mathbf{r}_{\mathrm{o}})}\mathbf{K}_{\pm} \times \mathbf{d}. \tag{54}$$

The dependence on the field point $\mathbf{r}$ enters as $\mathbf{r}-\mathbf{r}_{\mathrm{o}}$, which indicates that the field is emitted from the location $\mathbf{r}_{\mathrm{o}}$ of the dipole. From Eq. (51) we see that the image source function follows from the function $\mathbf{J}_-(\mathbf{k}_{\|})$ by reversing the sign of the parallel component. If we write $\mathbf{d}=\mathbf{d}_{\perp}+\mathbf{d}_{\|}$ for the dipole moment, then the image source function is

$$\mathbf{J}_+(\mathbf{k}_{\|})^{\mathrm{im}} = -i\omega\mathbf{d}^{\mathrm{im}}e^{-i\mathbf{K}_-\cdot\mathbf{r}_{\mathrm{o}}}, \tag{55}$$

with

$$\mathbf{d}^{\mathrm{im}} = \mathbf{d}_{\perp} - \mathbf{d}_{\|}, \tag{56}$$

the image dipole moment. We now introduce the image point of $\mathbf{r}_{\mathrm{o}}$ as

$$\mathbf{r}_{\mathrm{o}}^{\mathrm{im}} = \mathbf{r}_{\mathrm{o},\|} - \mathbf{r}_{\mathrm{o},\perp}, \tag{57}$$

which is $(x_{\mathrm{o}},y_{\mathrm{o}},-z_{\mathrm{o}})$ in Cartesian coordinates. With Eq. (7) we then have $\mathbf{K}_-\cdot\mathbf{r}_{\mathrm{o}}=\mathbf{K}_+\cdot\mathbf{r}_{\mathrm{o}}^{\mathrm{im}}$, and the reflected field becomes

$$\mathbf{B}(\mathbf{r})_{\mathrm{r}} = \frac{i\omega\mu_{\mathrm{o}}}{8\pi^2}\int \mathrm{d}^2\mathbf{k}_{\|}\frac{1}{\beta}e^{i\mathbf{K}_+\cdot(\mathbf{r}-\mathbf{r}_{\mathrm{o}}^{\mathrm{im}})}\mathbf{K}_+ \times \mathbf{d}^{\mathrm{im}}, \tag{58}$$

with Eq. (50). The $\mathbf{r}$ dependence enters here as $\mathbf{r}-\mathbf{r}_{\mathrm{o}}^{\mathrm{im}}$, which shows that this is the magnetic field emitted by a dipole with dipole moment $\mathbf{d}^{\mathrm{im}}$ located at $\mathbf{r}_{\mathrm{o}}^{\mathrm{im}}$.

Similarly, the current density (34) for a magnetic dipole $\mathbf{p}$ at $\mathbf{r}_{\mathrm{o}}$ gives the source functions

$$\mathbf{J}_{\pm}(\mathbf{k}_{\|}) = i\mathbf{K}_{\pm} \times \mathbf{p}e^{-i\mathbf{K}_{\pm}\cdot\mathbf{r}_{\mathrm{o}}}, \tag{59}$$

from which we obtain the image source function

$$\mathbf{J}_+(\mathbf{k}_{\|})^{\mathrm{im}} = i\mathbf{K}_+ \times \mathbf{p}^{\mathrm{im}}e^{-i\mathbf{K}_+\cdot\mathbf{r}_{\mathrm{o}}^{\mathrm{im}}}, \tag{60}$$

where

$$\mathbf{p}^{\mathrm{im}} = \mathbf{p}_{\|} - \mathbf{p}_{\perp} \tag{61}$$

is the image dipole moment. The reflected field is identical to the field by the image dipole $\mathbf{p}^{\mathrm{im}}$ located at the mirror position $\mathbf{r}_{\mathrm{o}}^{\mathrm{im}}$.

The case of an electric quadrupole $\overleftrightarrow{Q}$ located at $\mathbf{r}_{\mathrm{o}}$ is slightly more complicated. With the current density given by Eq. (40), we find for the source functions

$$\mathbf{J}_{\pm}(\mathbf{k}_{\|}) = -\frac{\omega}{6}\mathbf{K}_{\pm}\cdot\overleftrightarrow{Q}e^{-i\mathbf{K}_{\pm}\cdot\mathbf{r}_{\mathrm{o}}}. \tag{62}$$

The image source function then follows from the construction given in Eq. (51). We would like to write this as

$$\mathbf{J}_+(\mathbf{k}_{\|})^{\mathrm{im}} = -\frac{\omega}{6}\mathbf{K}_+\cdot\overleftrightarrow{Q}^{\mathrm{im}}e^{-i\mathbf{K}_+\cdot\mathbf{r}_{\mathrm{o}}^{\mathrm{im}}}, \tag{63}$$

so that $\overleftrightarrow{Q}^{\mathrm{im}}$ is the quadrupole moment of the image quadrupole at $\mathbf{r}_{\mathrm{o}}^{\mathrm{im}}$. To this end, we expand the quadrupole tensor on the Cartesian basis as

$$\overleftrightarrow{Q} = \sum_{\alpha,\beta} Q_{\alpha\beta}\mathbf{e}_{\alpha}\mathbf{e}_{\beta}, \quad \alpha,\beta = x,y,z, \tag{64}$$

and substitute this for $\overleftrightarrow{Q}$ in Eq. (62). Then we separate the perpendicular and parallel parts, apply Eq. (51), and write the result as in Eq. (63). The result for $\overleftrightarrow{Q}^{\mathrm{im}}$ can be written in dyadic form, as in Eq. (64), but more transparent is its matrix representation

$$\overleftrightarrow{Q}^{\mathrm{im}} \doteq \begin{bmatrix} -Q_{xx} & -Q_{xy} & Q_{xz} \\ -Q_{yx} & -Q_{yy} & Q_{yz} \\ Q_{zx} & Q_{zy} & -Q_{zz} \end{bmatrix} \tag{65}$$

on the Cartesian basis.

10. REFLECTED FIELD IN TERMS OF THE SOURCE CURRENT DENSITY

Equation (31) expresses the surface current density $\mathbf{i}(\mathbf{r})$ in the mirror directly in terms of the source current density $\mathbf{j}(\mathbf{r})$. The reflected field is expressed in terms of the source function $\mathbf{J}_-(\mathbf{k}_{\|})$ in Eq. (49) and in terms of the image source function $\mathbf{J}_+(\mathbf{k}_{\|})^{\mathrm{im}}$ in Eq. (50). In Section 9 it was shown that for multipoles the reflected field can be expressed as the field emitted by an image multipole located at $\mathbf{r}_{\mathrm{o}}^{\mathrm{im}}$ behind the mirror. We shall now show that the reflected field can also be expressed directly in terms of the source current density $\mathbf{j}(\mathbf{r})$.

The magnetic field generated by the surface current density is given by Eq. (2). Behind the mirror this field cancels exactly the incident field and in front of the mirror this is the reflected field. For $\mathbf{i}(\mathbf{r})$ in Eq. (2) we substitute the right-hand side of Eq. (31), which yields

$$\mathbf{B}(\mathbf{r})_{\mathrm{r}} = \frac{\mu_{\mathrm{o}}}{8\pi^2}\nabla\times\left[\mathbf{e}_z\times\int \mathrm{d}V'\mathbf{A}(\mathbf{r},\mathbf{r}')\times\mathbf{j}(\mathbf{r}')\right]. \quad (66)$$

Here we have introduced the universal vector function

$$\mathbf{A}(\mathbf{r},\mathbf{r}') = \int \mathrm{d}S'' g(\mathbf{r}-\mathbf{r}'')(\mathbf{r}''-\mathbf{r}')f(\mathbf{r}''-\mathbf{r}'). \quad (67)$$

Equation (66) expresses the reflected field for all $\mathbf{r}$ in front of the mirror as an integral over the source current density. With $(\mathbf{r}''-\mathbf{r}')f(\mathbf{r}''-\mathbf{r}')=\nabla'' g(\mathbf{r}''-\mathbf{r}')$, the integrand in Eq. (67) involves the Green's function twice. For these Green's functions we insert the angular spectrum representation (5), and use that both $\mathbf{r}$ and $\mathbf{r}'$ are in the region $z>0$. We then find

$$\mathbf{A}(\mathbf{r},\mathbf{r}') = -i\int \mathrm{d}^2\mathbf{k}_\parallel \frac{1}{\beta^2}\mathbf{K}_- e^{i\mathbf{K}_+\cdot\mathbf{r}} e^{-i\mathbf{K}_-\cdot\mathbf{r}'}, \quad z,z'>0, \quad (68)$$

as an angular spectrum representation for this function. The variable $\mathbf{r}'$ in Eq. (68) is the same as in Eq. (66), and therefore it represents a point in the source region. The mirror image of this point is $\mathbf{r}'^{\mathrm{im}}=\mathbf{r}'_\parallel-\mathbf{r}'_\perp$, when $\mathbf{r}'=\mathbf{r}'_\parallel+\mathbf{r}'_\perp$. We then have $\mathbf{K}_-\cdot\mathbf{r}'=\mathbf{K}_+\cdot\mathbf{r}'^{\mathrm{im}}$, and Eq. (68) can be written as

$$\mathbf{A}(\mathbf{r},\mathbf{r}') = -i\int \mathrm{d}^2\mathbf{k}_\parallel \frac{1}{\beta^2}\mathbf{K}_- e^{i\mathbf{K}_+\cdot(\mathbf{r}-\mathbf{r}'^{\mathrm{im}})}, \quad z,z'>0. \quad (69)$$

Here we observe that each partial wave has wave vector $\mathbf{K}_+$, so it corresponds to a wave propagating (or decaying) in the positive z direction, and the superposition appears to come from an image source behind the mirror.

As an example, let us consider again the electric dipole, for which the current density is given by Eq. (32). From Eq. (66) we find for the reflected field

$$\mathbf{B}(\mathbf{r})_{\mathrm{r}} = -\frac{i\omega\mu_{\mathrm{o}}}{8\pi^2}\nabla\times\{\mathbf{e}_z\times[\mathbf{A}(\mathbf{r},\mathbf{r}_{\mathrm{o}})\times\mathbf{d}]\}, \quad (70)$$

which is now expressed in terms of the dipole moment $\mathbf{d}$, rather than its image $\mathbf{d}^{\mathrm{im}}$.

11. CONCLUSIONS

When light reflects off a mirror, it induces a current density $\mathbf{i}(\mathbf{r})$ in its surface. An integral equation for the transformed current density $\mathbf{I}(\mathbf{k}_\parallel)$ was derived, and this equation has the incident electric field at the surface as an inhomogeneous term. The incident radiation is emitted by a source with a volume current density $\mathbf{j}(\mathbf{r})$, and when we adopt an angular spectrum representation of the field emitted by this source, the integral equation turns into an algebraic equation, which can subsequently be solved. The result is given by Eq. (26), which expresses the transformed current density $\mathbf{I}(\mathbf{k}_\parallel)$ in terms of the source function $\mathbf{J}_-(\mathbf{k}_\parallel)$. By transforming back to configuration space, we obtain relation (31). This equation relates the surface current density $\mathbf{i}(\mathbf{r})$ directly to the source current density $\mathbf{j}(\mathbf{r})$, and therefore $\mathbf{i}(\mathbf{r})$ can be calculated without the usual intermediate step of finding the magnetic field at the surface first. In Section 7 it was shown that the method allows for an easy evaluation of the current density induced by an electric or magnetic dipole, and by an electric quadrupole. The reflected field can be expressed in terms of the solution $\mathbf{I}(\mathbf{k}_\parallel)$, and in Section 8 it was shown that this leads immediately to the identification of the image source function, given by Eq. (51). As illustration, we have shown in Section 9 that this approach can be applied to the construction of images of multipoles. Alternatively, the reflected field can be expressed in terms of the source current density $\mathbf{j}(\mathbf{r})$, as shown in Eq. (66), and in this result any intermediate reference to the surface current density or to an image source has disappeared.

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