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Reflection off a mirror

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Electromagnetic radiation incident upon a perfect conductor induces a current density at the surface of the material. This surface current density is the solution of an integral equation, and it is shown that for a flat surface this equation has a very simple solution. The current density at a point of the surface is determined by the incident magnetic field at the same point. The surface current density generates an electromagnetic field, and the total radiation field is the sum of this field and the incident field. It is shown explicitly that inside the material the field of the current density cancels exactly the incident field. The reflected field can be expressed as an integral over the known surface current density, and is therefore determined by the incident magnetic field at the surface only. When the source of the incident field is known, the reflected field can be expressed in terms of a source function, which is determined by the current density of the source. It is shown that this approach leads to a simple way of constructing the mirror image of an arbitrary source, and we illustrate the concept by determining the mirror image of an electric quadrupole.

1. Introduction

Scattering of electromagnetic radiation by an object or a structured surface of infinite extent has been the subject of numerous studies. The simplest case is the reflection and transmission of a plane polarized wave by a flat semi-infinite medium, or a by an infinite layer. The reflected and transmitted waves are again plane polarized waves, and their amplitudes are the Fresnel coefficients [1]. The problem of scattering of a plane wave by a dielectric or metallic sphere (Mie theory) is another example of a problem that can be solved analytically [2, 3]. When the object or the surface is more complex, one usually has to resort to numerical methods. In the differential equation approach the incident field, the scattered field, and the field in the material are expanded onto a complete set of functions. Then boundary conditions are imposed, which leads to a set of linear equations for the expansion coefficients, and this set is solved numerically [4–6]. In the integral equation approach, the scattered field and the field inside the material are a solution of a linear integral equation, with the incident field as the inhomogeneous term. The field inside the material and the scattered field are expanded onto a complete set of functions,

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which then leads to a set of linear equations for the expansion coefficients. This is known as the Method of Moments [7]. There exist many variations on this approach [8], because of the existence of a variety of integrals of Maxwell's equations. Particularly interesting is the case where the object or the surface is a perfect conductor. Then there is no field inside the material, so only the scattered field has to be considered. Scattering of a plane wave by a perfectly conducting object or surface has been studied for a large array of geometries [9–17].

A problem of a different nature arises when the incident field is not a plane wave, but radiation emitted by a particular source. For example, when an electric dipole is located near a slab of dielectric or metallic material, then part of the radiation reflects and part of it is transmitted by the layer. Dipole radiation contains both travelling and evanescent waves, and upon transmission an evanescent wave can be converted into a travelling wave, and vice versa. This problem can be solved in closed form [18], and the method can be generalized to multipole fields of arbitrary order [19, 20]. The method of solution relies on an angular spectrum representation of the multipole fields. Such a representation is a superposition of plane waves, and the reflection and transmission of each plane wave is accounted for by Fresnel coefficients. For a multipole near a perfect conductor, the reflected field can also be obtained by the method of images [21]. Here we consider the reflection of an arbitrary incident field by a flat perfectly-conducting surface (mirror), and we show that this problem can be solved in closed form. When the source of the incident field is known, this solution also provides a method for obtaining the mirror image of this source.

2. Reflection off a perfect conductor

Let us first consider an electromagnetic field, incident upon the surface of a perfect conductor, as shown in figure 1. Inside the perfect conductor there is no field and no current density. The only current appears as a surface current density $\boldsymbol{i}(\boldsymbol{r})$, which is a vector in the local tangent plane. We assume a harmonic time dependence with angular frequency ω so that $\boldsymbol{i}(\boldsymbol{r})$ is the complex amplitude of $\boldsymbol{i}(\boldsymbol{r}, t)$, e.g. $\boldsymbol{i}(\boldsymbol{r}, t) = \mathrm{Re}[\boldsymbol{i}(\boldsymbol{r})\exp(-i\omega t)]$, and similarly for other time dependent fields. The vector potential, or Hertz vector, for $\boldsymbol{i}(\boldsymbol{r})$ is

$$\boldsymbol{\Pi}(\boldsymbol{r}) = \frac{\mu_o}{4\pi}\int \mathrm{d}S'\boldsymbol{i}(\boldsymbol{r}')g(\boldsymbol{r}-\boldsymbol{r}'), \tag{1}$$

with

$$g(\boldsymbol{r}-\boldsymbol{r}') = \frac{e^{ik_o|\boldsymbol{r}-\boldsymbol{r}'|}}{|\boldsymbol{r}-\boldsymbol{r}'|}, \tag{2}$$

the Green's function for the scalar Helmholtz equation, and $k_o = \omega/c$. The integral in equation (1) runs over the entire surface. The magnetic field generated by the surface

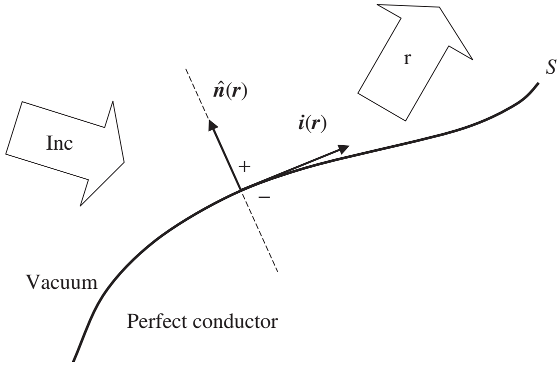

Figure 1. Surface S is the boundary between vacuum and a perfectly conducting metal. An incident field induces a current density $\boldsymbol{i}(\boldsymbol{r})$ on the surface. The current density generates an electromagnetic field, which is the reflected field in vacuum, and in the material this field cancels exactly the incident field, so that there is no field inside the material. The current density on the surface is a solution of equation (8), which has the incident magnetic field as inhomogeneous term.

current density is $\nabla \times \boldsymbol{\Pi}(\boldsymbol{r})$, and the total magnetic field is

$$\boldsymbol{B}(\boldsymbol{r}) = \boldsymbol{B}(\boldsymbol{r})_{\text{inc}} + \nabla \times \boldsymbol{\Pi}(\boldsymbol{r}), \tag{3}$$

with $\boldsymbol{B}(\boldsymbol{r})_{\text{inc}}$ the incident magnetic field. Explicitly,

$$\boldsymbol{B}(\boldsymbol{r}) = \boldsymbol{B}(\boldsymbol{r})_{\text{inc}} - \frac{\mu_o}{4\pi}\int \mathrm{d}S' \boldsymbol{i}(\boldsymbol{r}') \times \nabla g(\boldsymbol{r}-\boldsymbol{r}'). \tag{4}$$

Equations (3) and (4) hold for any point $\boldsymbol{r}$ not on the boundary surface S. For a field point $\boldsymbol{r}$ in the vacuum, the reflected field is

$$\boldsymbol{B}(\boldsymbol{r})_{\text{r}} = \nabla \times \boldsymbol{\Pi}(\boldsymbol{r}). \tag{5}$$

On the other hand, for a field point $\boldsymbol{r}$ in the material we have $\boldsymbol{B}(\boldsymbol{r}) = 0$, and therefore the field generated by $\boldsymbol{i}(\boldsymbol{r})$ must cancel exactly the incident field. When we move the field point $\boldsymbol{r}$ across S from the material to the vacuum, the magnetic field jumps from zero to some finite value. This discontinuity comes from the integral on the right-hand side of equation (4). Let us indicate by $\boldsymbol{r}_+$ and $\boldsymbol{r}_-$ field points just outside and inside the medium, and near point $\boldsymbol{r}$ in S. With a limit procedure it can be shown using equation (4) that [22]

$$\boldsymbol{B}(\boldsymbol{r}_\pm) = \boldsymbol{B}(\boldsymbol{r})_{\text{inc}} \pm \frac{1}{2}\mu_o \boldsymbol{i}(\boldsymbol{r}) \times \hat{\boldsymbol{n}}(\boldsymbol{r}) - \frac{\mu_o}{4\pi} P \int \mathrm{d}S' \boldsymbol{i}(\boldsymbol{r}') \times \nabla g(\boldsymbol{r}-\boldsymbol{r}'), \tag{6}$$

where $\hat{\boldsymbol{n}}(\boldsymbol{r})$ is the unit normal on S at $\boldsymbol{r}$, directed from the material to the vacuum. For the integral on the right-hand side we take point $\boldsymbol{r}$ in S. The factor $\nabla g(\boldsymbol{r}-\boldsymbol{r}')$ in the integrand of the integral has a singularity at $\boldsymbol{r}' = \boldsymbol{r}$, and it is understood that we leave out a small circle around $\boldsymbol{r}$, and that in the end we let the radius of this circle shrink to zero. In this sense, the integral is a principal value integral, which is indicated by P. This integral is the same for $\boldsymbol{B}(\boldsymbol{r}_+)$ and $\boldsymbol{B}(\boldsymbol{r}_-)$, so for the difference we obtain

$$\boldsymbol{B}(\boldsymbol{r}_+) - \boldsymbol{B}(\boldsymbol{r}_-) = \mu_o \boldsymbol{i}(\boldsymbol{r}) \times \hat{\boldsymbol{n}}(\boldsymbol{r}), \tag{7}$$

and this is the usual boundary condition for the magnetic field at an interface. Therefore, this boundary condition is automatically satisfied by equation (4), no matter what the surface current density is. Then, inside the perfect conductor the magnetic field is zero, so we have $\boldsymbol{B}(\boldsymbol{r}_-) = 0$. From equation (6) we then find

$$\tfrac{1}{2}\mu_o \boldsymbol{i}(\boldsymbol{r}) \times \hat{\boldsymbol{n}}(\boldsymbol{r}) + \frac{\mu_o}{4\pi} P \int \mathrm{d}S' \boldsymbol{i}(\boldsymbol{r}') \times \nabla g(\boldsymbol{r} - \boldsymbol{r}') = \boldsymbol{B}(\boldsymbol{r})_{\mathrm{inc}}. \tag{8}$$

Given the incident field, this is a linear integral equation for $\boldsymbol{i}(\boldsymbol{r})$, with $\boldsymbol{B}(\boldsymbol{r})_{\mathrm{inc}}$ as the inhomogeneous term. In a numerical approach, this equation is solved with the Method of Moments. After solving, $\boldsymbol{\Pi}(\boldsymbol{r})$ is computed with equation (1), and then the reflected field follows by taking the curl, according to equation (5). Finally, the reflected electric field follows from a Maxwell equation:

$$\boldsymbol{E}(\boldsymbol{r})_{\mathrm{r}} = \frac{ic^2}{\omega} \nabla \times \boldsymbol{B}(\boldsymbol{r})_{\mathrm{r}}. \tag{9}$$

3. Current density in a mirror

Equation (8) can only be solved for $\boldsymbol{i}(\boldsymbol{r})$ analytically if the shape of the surface is simple enough. For instance, it can be solved in closed form for a sphere in terms of vector spherical harmonics. Another example, which we shall consider here, is a flat surface. The gradient of the Green's function is

$$\nabla g(\boldsymbol{r} - \boldsymbol{r}') = (\boldsymbol{r} - \boldsymbol{r}') \left(\frac{ik_o}{|\boldsymbol{r} - \boldsymbol{r}'|^2} - \frac{1}{|\boldsymbol{r} - \boldsymbol{r}'|^3} \right) e^{ik_o|\boldsymbol{r} - \boldsymbol{r}'|}. \tag{10}$$

Let the surface be the xy-plane, and let the material occupy the region $z < 0$. The integration in equation (8) runs over the xy-plane, so the point $\boldsymbol{r}'$ is in the xy-plane. For the principal value integral, point $\boldsymbol{r}$ is also in the xy-plane, and therefore the vector $\boldsymbol{r} - \boldsymbol{r}'$ lies in the xy-plane, and so does $\nabla g(\boldsymbol{r} - \boldsymbol{r}')$. Since the current density $\boldsymbol{i}(\boldsymbol{r}')$ is also in the xy-plane we see that $\boldsymbol{i}(\boldsymbol{r}') \times \nabla g(\boldsymbol{r} - \boldsymbol{r}')$ is along the z-axis. Consequently, the principal value integral in equation (8) is a vector along the z-axis. We now take the cross product of equation (8) with $\boldsymbol{e}_z$. The contribution from the integral vanishes, and $\boldsymbol{e}_z \times [\boldsymbol{i}(\boldsymbol{r}) \times \hat{\boldsymbol{n}}(\boldsymbol{r})] = \boldsymbol{i}(\boldsymbol{r})$, since $\hat{\boldsymbol{n}}(\boldsymbol{r}) = \boldsymbol{e}_z$. Equation (8) becomes

$$\boldsymbol{i}(\boldsymbol{r}) = \frac{2}{\mu_o} \boldsymbol{e}_z \times \boldsymbol{B}(\boldsymbol{r})_{\mathrm{inc}}, \tag{11}$$

which is the explicit solution for $\boldsymbol{i}(\boldsymbol{r})$, given the incident magnetic field. Apparently, for a flat surface the current density at point $\boldsymbol{r}$ on the surface only depends on the value of the incident magnetic field at that same point. In the general case of equation (8), the current density at $\boldsymbol{r}$ depends on the current density at every other point $\boldsymbol{r}'$ through the principal value integral. Interesting to note is that if we set $\boldsymbol{B}(\boldsymbol{r}_-)=0$ in the boundary condition (7), and then take the cross product with $\boldsymbol{e}_z$ we find

$$\boldsymbol{i}(\boldsymbol{r}) = \frac{1}{\mu_o}\boldsymbol{e}_z \times \boldsymbol{B}(\boldsymbol{r}_+). \tag{12}$$

Here, $\boldsymbol{B}(\boldsymbol{r}_+)$ is the total field just outside the medium, which is the sum of the incident field and the reflected field. The additional factor of 2 in equation (11) indicates that the incident field and the reflected field contribute equally to $\boldsymbol{i}(\boldsymbol{r})$ at point $\boldsymbol{r}$. From equation (6) with $\boldsymbol{B}(\boldsymbol{r}_-)=0$ we have

$$\boldsymbol{B}(\boldsymbol{r}_+) = \mu_o \boldsymbol{i}(\boldsymbol{r}) \times \boldsymbol{e}_z, \tag{13}$$

so $\boldsymbol{B}(\boldsymbol{r}_+)$ is a vector in the xy-plane. The incident and the reflected field will both have a component along the z-axis at $z=0^+$, and these components cancel exactly.

4. The magnetic field

With the current density in the mirror given by equation (11), the vector potential (1) becomes

$$\boldsymbol{\Pi}(\boldsymbol{r}) = \frac{1}{2\pi}\boldsymbol{e}_z \times \int \mathrm{d}S' \boldsymbol{B}(\boldsymbol{r}')_{\mathrm{inc}} g(\boldsymbol{r}-\boldsymbol{r}'). \tag{14}$$

Taking the curl and simplifying with a vector identity then yields for the magnetic field

$$\begin{aligned}\boldsymbol{B}(\boldsymbol{r}) = \boldsymbol{B}(\boldsymbol{r})_{\mathrm{inc}} &- \frac{1}{2\pi}\int \mathrm{d}S'\left[\boldsymbol{B}(\boldsymbol{r}')_{\mathrm{inc}}\frac{\partial}{\partial z}g(\boldsymbol{r}-\boldsymbol{r}')\right] \\ &+ \frac{1}{2\pi}\boldsymbol{e}_z \int \mathrm{d}S'[\boldsymbol{B}(\boldsymbol{r}')_{\mathrm{inc}} \cdot \nabla g(\boldsymbol{r}-\boldsymbol{r}')].\end{aligned} \tag{15}$$

Since the integrals run over the xy-plane, this shows explicitly that the total magnetic field off the xy-plane is determined completely by the given value of the incident field $\boldsymbol{B}(\boldsymbol{r})_{\mathrm{inc}}$ in the xy-plane. For $z>0$, the sum of the second and the third term on the right-hand side is the reflected field. For $z<0$, the three terms on the right-hand side should add up to zero, since the magnetic field in the material is zero. We shall show this in section 6.

The magnetic field can also be expressed in terms of an angular spectrum, which is particularly useful if the source of the incident radiation is known. We adopt Weyl's representation of the Green's function [23]

$$g(\boldsymbol{r}-\boldsymbol{r}') = \frac{i}{2\pi}\int \mathrm{d}^2\boldsymbol{k}_{||}\frac{1}{\beta}e^{i\boldsymbol{k}_{||}\cdot(\boldsymbol{r}-\boldsymbol{r}')+i\beta|z-z'|}. \tag{16}$$

Vector $\boldsymbol{k}_{\|}$ is a vector in the xy-plane, and the integration is a surface integral over the $\boldsymbol{k}_{\|}$ plane. The parameter β is defined by

$$\beta = \begin{cases} \sqrt{k_o^2 - k_{\|}^2}, & k_{\|} < k_o \\ i\sqrt{k_{\|}^2 - k_o^2}, & k_{\|} > k_o \end{cases}, \tag{17}$$

with $k_{\|}$ the magnitude of $\boldsymbol{k}_{\|}$. For $k_{\|} < k_o$, β is real and the partial wave is travelling, whereas for $k_{\|} > k_o$ the partial wave is evanescent. These waves decay exponentially away from the plane $z = z'$. We substitute the expression (16) into equation (15), with $z' = 0$ since $\boldsymbol{r}'$ is in the xy-plane, and work out the derivatives. We then obtain the alternative representation for the magnetic field near the mirror

$$\boldsymbol{B}(\boldsymbol{r}) = \boldsymbol{B}(\boldsymbol{r})_{\text{inc}} + \frac{1}{4\pi^2}\int \mathrm{d}^2\boldsymbol{k}_{\|}\frac{1}{\beta}\langle \beta\,\mathrm{sgn}(z)\tilde{\boldsymbol{B}}(\boldsymbol{k}_{\|})_{\text{inc}} - \boldsymbol{e}_z\{\tilde{\boldsymbol{B}}(\boldsymbol{k}_{\|})_{\text{inc}}\cdot[\boldsymbol{k}_{\|} + \beta\,\mathrm{sgn}(z)\boldsymbol{e}_z]\}\rangle e^{i\boldsymbol{k}_{\|}\cdot\boldsymbol{r} + i\beta|z|}, \tag{18}$$

with

$$\tilde{\boldsymbol{B}}(k_{\|})_{\text{inc}} = \int \mathrm{d}S'\boldsymbol{B}(\boldsymbol{r}')_{\text{inc}}e^{-i\boldsymbol{k}_{\|}\cdot\boldsymbol{r}'}, \tag{19}$$

the spatial Fourier transform of $\boldsymbol{B}(\boldsymbol{r})_{\text{inc}}$ in the xy-plane.

Expression (18) holds for both $z > 0$ and $z < 0$. For $z > 0$, we have $\mathrm{sgn}(z) = 1$, and with the notation

$$\boldsymbol{K}_{\pm} = \boldsymbol{k}_{\|} \pm \beta\boldsymbol{e}_z, \tag{20}$$

the result (18) can be simplified to

$$\boldsymbol{B}(\boldsymbol{r}) = \boldsymbol{B}(\boldsymbol{r})_{\text{inc}} - \frac{1}{4\pi^2}\int \mathrm{d}^2\boldsymbol{k}_{\|}\frac{1}{\beta}e^{i\boldsymbol{K}_+\cdot\boldsymbol{r}}\boldsymbol{K}_+ \times [\boldsymbol{e}_z \times \tilde{\boldsymbol{B}}(\boldsymbol{k}_{\|})_{\text{inc}}], \quad z > 0. \tag{21}$$

The second term on the right-hand side is the reflected field. Similarly, for $z < 0$ we can write equation (18) as

$$\boldsymbol{B}(\boldsymbol{r}) = \boldsymbol{B}(\boldsymbol{r})_{\text{inc}} - \frac{1}{4\pi^2}\int \mathrm{d}^2\boldsymbol{k}_{\|}e^{i\boldsymbol{K}_-\cdot\boldsymbol{r}}\tilde{\boldsymbol{B}}(\boldsymbol{k}_{\|})_{\text{inc}} - \frac{1}{4\pi^2}\boldsymbol{e}_z\int \mathrm{d}^2\boldsymbol{k}_{\|}\frac{1}{\beta}e^{i\boldsymbol{K}_-\cdot\boldsymbol{r}}[\boldsymbol{K}_-\cdot\tilde{\boldsymbol{B}}(\boldsymbol{k}_{\|})_{\text{inc}}], \quad z < 0, \tag{22}$$

and this should be zero.

5. Source of the incident field

Equations (21) and (22) give the magnetic field for an arbitrary incident field $\boldsymbol{B}(\boldsymbol{r})_{\text{inc}}$. However, we have not used any property of this field, and in particular that it has

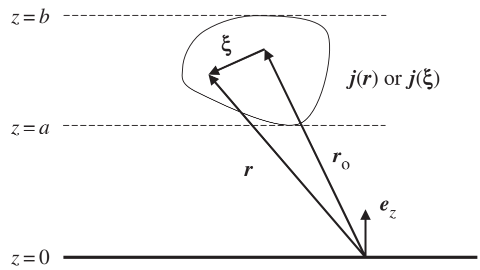

Figure 2. The source of the incident radiation is the current density $\boldsymbol{j}(\boldsymbol{r})$, which is located in the region $a<z<b$. For a localized source around some point $\boldsymbol{r}_o$, it is advantageous to use the position vector $\boldsymbol{\xi}$ to indicate a point in the source distribution.

to satisfy Maxwell's equations, when combined with the corresponding electric field. Let us now consider the situation where the incident field is emitted by a localized source in the region $z>0$, as shown in figure 2. The source has a given current density $\boldsymbol{j}(\boldsymbol{r})$, and the radiated magnetic field is the curl of the vector potential, as in equation (4), but now the integral runs over the volume of the source. We have for the field generated by the source

$$\boldsymbol{B}(\boldsymbol{r}) = -\frac{\mu_o}{4\pi}\int \mathrm{d}^3\boldsymbol{r}'\boldsymbol{j}(\boldsymbol{r}') \times \nabla g(\boldsymbol{r}-\boldsymbol{r}'). \tag{23}$$

In order to obtain an angular spectrum representation of this expression, we substitute Weyl's representation (16) for the Green's function. To simplify the appearance of $|z-z'|$ in the Green's function, we shall only consider the solutions in $z>b$ and $z<a$ in the notation of figure 2. For $z>b$ we have $|z-z'|=z-z'$, since the value of the integration variable z' is restricted to $a<z'<b$, and similarly for $z<a$ we have $|z-z'|=-(z-z')$. With equation (20) we have

$$\boldsymbol{k}_{\|}\cdot(\boldsymbol{r}-\boldsymbol{r}')+\beta|z-z'| = \boldsymbol{K}_{\pm}\cdot(\boldsymbol{r}-\boldsymbol{r}'), \tag{24}$$

where the upper (lower) sign refers to the region $z>b$ $(z<a)$. Then we define the two source functions

$$\boldsymbol{D}(\boldsymbol{k}_{\|})_{\pm} = \frac{i}{\omega}\int \mathrm{d}^3\boldsymbol{r}\boldsymbol{j}(\boldsymbol{r})e^{-i\boldsymbol{K}_{\pm}\cdot\boldsymbol{r}}, \tag{25}$$

in terms of which the magnetic field of the source can be represented as

$$\boldsymbol{B}(\boldsymbol{r}) = \frac{i\omega\mu_o}{8\pi^2}\int \mathrm{d}^2\boldsymbol{k}_{\|}\frac{1}{\beta}e^{i\boldsymbol{K}_{\pm}\cdot\boldsymbol{r}}\boldsymbol{k}_{\pm}\times\boldsymbol{D}(\boldsymbol{k}_{\|})_{\pm}. \tag{26}$$

Here again the upper (lower) sign refers to the region $z>b$ $(z<a)$. This is a superposition of plane waves, and each partial wave has a wavevector $\boldsymbol{K}_{\pm}=\boldsymbol{K}_{\|}\pm\beta\boldsymbol{e}_z$. The z-component is $\pm\beta$. Therefore, the solution in $z>b$ travels away from the source region towards larger z values for β real, and decays away from the source in the positive z direction for β imaginary. Similarly, the solution in the region $z<a$ travels or decays towards the xy-plane. The solution with the lower sign in equation (26) is the incident field on the mirror, so we have explicitly

$$\boldsymbol{B}(\boldsymbol{r})_{\rm inc} = \frac{i\omega\mu_{\rm o}}{8\pi^2}\int {\rm d}^2\boldsymbol{k}_{||}\frac{1}{\beta}e^{i\boldsymbol{K}_-\cdot\boldsymbol{r}}\boldsymbol{K}_- \times \boldsymbol{D}(\boldsymbol{k}_{||})_-. \tag{27}$$

6. Magnetic field in terms of the source function $\boldsymbol{D}(\boldsymbol{k}_{||})_-$

Equation (27) expresses the incident field in terms of the source function $\boldsymbol{D}(\boldsymbol{k}_{||})_-$, whereas in equation (21) the reflected field is expressed in terms of the Fourier transform $\tilde{\boldsymbol{B}}(\boldsymbol{k}_{||})_{\rm inc}$ of the incident field in the xy-plane. We shall now derive a relation between these two functions of $\boldsymbol{k}_{||}$. The incident field in the xy-plane follows from setting $z=0$ in equation (27), which gives

$$\boldsymbol{B}(\boldsymbol{r}')_{\rm inc} = \frac{i\omega\mu_{\rm o}}{8\pi^2}\int {\rm d}^2\boldsymbol{k}_{||}\frac{1}{\beta}e^{i\boldsymbol{k}_{||}\cdot\boldsymbol{r}'}\boldsymbol{K}_- \times \boldsymbol{D}(\boldsymbol{k}_{||})_- \quad , \quad xy-\text{plane}. \tag{28}$$

Then we multiply by $\exp(-i\boldsymbol{k}'_{||}\cdot\boldsymbol{r}')$, as in equation (19), and we integrate over the xy-plane. With the spectral representation of the two-dimensional delta function

$$\int {\rm d}S' e^{i(\boldsymbol{k}_{||}-\boldsymbol{k}'_{||})\cdot\boldsymbol{r}'} = 4\pi^2\delta(\boldsymbol{k}_{||}-\boldsymbol{k}'_{||}), \tag{29}$$

we obtain the simple relation

$$\tilde{\boldsymbol{B}}(\boldsymbol{k}_{||})_{\rm inc} = \frac{i\omega\mu_{\rm o}}{2\beta}\boldsymbol{K}_- \times \boldsymbol{D}(\boldsymbol{k}_{||})_-. \tag{30}$$

When we substitute this into equation (21) we find

$$\boldsymbol{B}(\boldsymbol{r}) = \boldsymbol{B}(\boldsymbol{r})_{\rm inc} - \frac{i\omega\mu_{\rm o}}{8\pi^2}\int {\rm d}^2\boldsymbol{k}_{||}\frac{1}{\beta^2}e^{i\boldsymbol{K}_+\cdot\boldsymbol{r}}\boldsymbol{K}_+ \times \left\{\boldsymbol{e}_z \times \left[\boldsymbol{K}_- \times \boldsymbol{D}(\boldsymbol{k}_{||})_-\right]\right\}, \tag{31}$$

for the magnetic field. The angular spectrum representation of $\boldsymbol{B}(\boldsymbol{r})_{\rm inc}$ is given by equation (27), which holds for the region $0<z<a$. The second term on the right-hand side of equation (31) is the reflected field, and this representation holds of course for all $z>0$.

Next we consider the field in $z<0$, given by equation (22). From equation (30) we have $\boldsymbol{K}_-\cdot\tilde{\boldsymbol{B}}(\boldsymbol{k}_{||})_{\rm inc}=0$, and therefore the last term on the right-hand side of equation (22) vanishes. When we combine equations (27) and (30) we have

$$\boldsymbol{B}(\boldsymbol{r})_{\rm inc} = \frac{1}{4\pi^2}\int {\rm d}^2\boldsymbol{k}_{||}e^{i\boldsymbol{K}_-\cdot\boldsymbol{r}}\tilde{\boldsymbol{B}}(\boldsymbol{k}_{||})_{\rm inc}. \tag{32}$$

This shows that the incident field $\boldsymbol{B}(\boldsymbol{r})_{\rm inc}$ for all $\boldsymbol{r}$ in $z<a$ is determined by knowledge of this field in the xy-plane only, since the evaluation of $\tilde{\boldsymbol{B}}(\boldsymbol{k}_{||})_{\rm inc}$ only involves the field in the xy-plane. From equation (32) we see that the first integral on the

right-hand side of equation (22) is just $\boldsymbol{B}(\boldsymbol{r})_{\text{inc}}$, and therefore the first two terms in equation (22) cancel. This proves that $\boldsymbol{B}(\boldsymbol{r})=0$ for $z<0$.

7. Verification of the boundary condition

Our solution (31) for the magnetic field in $0<z<a$ was obtained by solving the integral equation (8) for the current density, after which the field was expressed as the sum of the incident field and the field radiated by $\boldsymbol{i}(\boldsymbol{r})$. In a more conventional approach to reflection of radiation by an interface, like for the computation of the Fresnel coefficients for a layer, one writes down general solutions of Maxwell's equations at both sides of the interface, containing some undetermined constants. For the layer problem, these are the Fresnel coefficients. These constants are then evaluated by imposing the boundary conditions for the electric and magnetic fields at the interface. We now show that for our solution the boundary condition for the magnetic field is satisfied.

The general boundary condition for $\boldsymbol{B}(\boldsymbol{r})$ is that its normal component must be continuous across the boundary, which follows from Maxwell's equation $\nabla\cdot\boldsymbol{B}(\boldsymbol{r})=0$. For a perfect conductor we have $\boldsymbol{B}(\boldsymbol{r})=0$ inside the material, and therefore it must hold that the normal component of $\boldsymbol{B}(\boldsymbol{r})$ is zero just outside the medium, as also follows from equation (13). With $\boldsymbol{r}_+$ a point just outside the material, we must therefore show that $\boldsymbol{e}_z\cdot\boldsymbol{B}(\boldsymbol{r}_+)=0$. Since the z-component of $\boldsymbol{r}_+$ is zero in the limit where this point approaches the surface, we have $\boldsymbol{K}_\pm\cdot\boldsymbol{r}_+=\boldsymbol{k}_\| . \boldsymbol{r}_+$ from equation (20). With this observation, equations (27) and (31) can be combined as

$$\boldsymbol{B}(\boldsymbol{r}_+)=\frac{i\omega\mu_o}{8\pi^2}\int d^2\boldsymbol{k}_\|\frac{1}{\beta^2}e^{i\boldsymbol{k}_\|\cdot\boldsymbol{r}_+}\left\langle\beta\boldsymbol{K}_-\times\boldsymbol{D}(\boldsymbol{k}_\|)_--\boldsymbol{K}_+\times\left\{\boldsymbol{e}_z\times\left[\boldsymbol{K}_-\times\boldsymbol{D}(\boldsymbol{k}_\|)_-\right]\right\}\right\rangle. \quad (33)$$

From equation (20) we derive

$$\boldsymbol{K}_+\times(\boldsymbol{e}_z\times\boldsymbol{a})=(\boldsymbol{K}_-\cdot\boldsymbol{a})\boldsymbol{e}_z+2\beta\boldsymbol{e}_z(\boldsymbol{e}_z\cdot\boldsymbol{a})-\beta\boldsymbol{a}, \quad (34)$$

for any vector $\boldsymbol{a}$. For $\boldsymbol{a}=\boldsymbol{K}_-\times\boldsymbol{D}(\boldsymbol{k}_\|)_-$, the first term on the right-hand side vanishes, and we have

$$\boldsymbol{K}_+\times\{\boldsymbol{e}_z\times[\boldsymbol{K}_-\times\boldsymbol{D}(\boldsymbol{k}_{||})_-]\}=2\beta\boldsymbol{e}_z\{\boldsymbol{e}_z\cdot[\boldsymbol{K}_-\times\boldsymbol{D}(\boldsymbol{k}_{||})_-]\}-\beta\boldsymbol{K}_-\times\boldsymbol{D}(\boldsymbol{k}_{||})_-. \quad (35)$$

Taking the dot product with $\boldsymbol{e}_z$ yields

$$\boldsymbol{e}_z\cdot\left\langle\boldsymbol{K}_+\times\{\boldsymbol{e}_z\times[\boldsymbol{K}_-\times\boldsymbol{D}(\boldsymbol{k}_{||})_-]\}\right\rangle=\beta\boldsymbol{e}_z\cdot[\boldsymbol{K}_-\times\boldsymbol{D}(\boldsymbol{k}_{||})_-], \quad (36)$$

and this gives $\boldsymbol{e}_z\cdot\boldsymbol{B}(\boldsymbol{r}_+)=0$, which is the boundary condition.

8. Image source

In the method of images [24] the mirror is replaced by an image source in $z<0$, in such a way that the field radiated by the image source into the region $z>0$ is the same as the reflected field in the problem with the mirror. We now construct the image of an arbitrary source, as in figure 2, and determine the corresponding source function for the angular spectrum representation of the image field. Then we prove that the field in $z>0$ of the image source is equal to the reflected field $\boldsymbol{B}(\boldsymbol{r})_{\mathrm{r}}$ by the mirror.

As usual in the method of images, we first guess what the image source is, and then we prove afterwards that it is correct. Let $\boldsymbol{r}$ be a position vector of a point in the source, and we write $\boldsymbol{r}=\boldsymbol{r}_{\parallel}+\boldsymbol{r}_{\perp}$, where the subscripts $\parallel$ and $\perp$ refer to the parallel and perpendicular components with respect to the xy-plane, respectively. The mirror image of this point is

$$\boldsymbol{r}^{\mathrm{im}}=\boldsymbol{r}_{\parallel}-\boldsymbol{r}_{\perp}, \tag{37}$$

and the mirror image of a positive charge is a negative charge, as indicated in figure 3(a). When this positive charge moves along the xy-plane, as in figure 3(b), its mirror image moves in the same direction, but since this is a negative charge, the corresponding current density is in the opposite direction. When the positive charge moves away from the xy-plane, as in figure 3(c), its negative mirror image moves in the negative z-direction, corresponding to a current density in the positive z-direction. Therefore, if we write $\boldsymbol{j}(\boldsymbol{r})=\boldsymbol{j}(\boldsymbol{r})_{\perp}+\boldsymbol{j}(\boldsymbol{r})_{\parallel}$ for the current density at point $\boldsymbol{r}$ in the source, then the current density at the mirror position $\boldsymbol{r}^{\mathrm{im}}$ is

$$\boldsymbol{j}(\boldsymbol{r}^{\mathrm{im}})=\boldsymbol{j}(\boldsymbol{r})_{\perp}-\boldsymbol{j}(\boldsymbol{r})_{\parallel}. \tag{38}$$

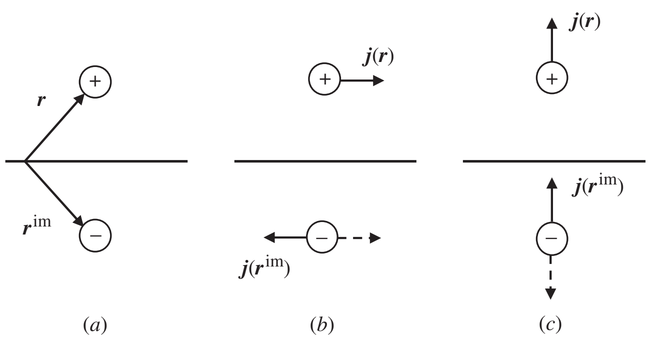

Figure 3. (a) The mirror image of a point with position vector $\boldsymbol{r}$ is the point with position vector $\mathbf{r}^{\mathrm{im}}$, and the mirror image of a positive charge is a negative charge, as is illustrated. (b) When the positive charge moves parallel to the surface, its mirror image moves in the same direction, but since this is a negative charge, the current density of the image source at the image position is in the opposite direction. (c) When the positive charge moves away from the surface, so does the mirror image. The corresponding current density of the image source at the image position is therefore the same as the current density of the source. These considerations lead to equation (38) for the current density of the image source.

The angular spectrum representation of the field radiated by the image source is identical in form as the field generated by $\boldsymbol{j}(\boldsymbol{r})$, with the explicit form given by equation (26). The shape of the image source is the mirror image of the source, since for every $\boldsymbol{r}$ in the source there is a corresponding $\boldsymbol{r}^{\text{im}}$ in the image source. Therefore the region $a<z<b$ in figure 2 has a mirror image region $-b<z<-a$, and for the field radiated by the image source in $z>0$ we need the solution for $z>-a$, which is the upper sign in equation (26). The source function for the image source is

$$\boldsymbol{D}(\boldsymbol{k}_{||})_{+}^{\text{im}} = \frac{i}{\omega}\int \mathrm{d}^3\boldsymbol{r}^{\text{im}}\boldsymbol{j}(\boldsymbol{r}^{\text{im}})e^{-i\boldsymbol{K}_{+}\cdot\boldsymbol{r}^{\text{im}}}, \tag{39}$$

as in equation (25) (we will not need the – source function here), and the image field in $z>-a$ is

$$\boldsymbol{B}(\boldsymbol{r})_{\text{im}} = \frac{i\omega\mu_{\text{o}}}{8\pi^2}\int \mathrm{d}^2\boldsymbol{k}_{||}\frac{1}{\beta}e^{i\boldsymbol{K}_{+}\cdot\boldsymbol{r}}\boldsymbol{K}_{+}\times\boldsymbol{D}(\boldsymbol{k}_{||})_{+}^{\text{im}}. \tag{40}$$

With equations (20) and (37) we have $\boldsymbol{K}_{+}\cdot\boldsymbol{r}^{\text{im}}=\boldsymbol{K}_{-}\cdot\boldsymbol{r}$, and for $\boldsymbol{j}(\boldsymbol{r}^{\text{im}})$ we substitute the right-hand side of equation (38) in the integrand of the integral in equation (39). For $\mathrm{d}^3\boldsymbol{r}^{\text{im}}$ we write $\mathrm{d}^3\boldsymbol{r}$, since this is just a symbolic notation for the volume element. Then equation (39) becomes

$$\boldsymbol{D}(\boldsymbol{k}_{||})_{+}^{\text{im}} = \frac{i}{\omega}\int \mathrm{d}^3\boldsymbol{r}[\boldsymbol{j}(\boldsymbol{r})_{\perp} - \boldsymbol{j}(\boldsymbol{r})_{||}]e^{-i\boldsymbol{K}_{-}\cdot\boldsymbol{r}}, \tag{41}$$

and the integral now runs over the source, rather than over the image source. Comparison with equation (25) then gives

$$\boldsymbol{D}(\boldsymbol{k}_{||})_{+}^{\text{im}} = \boldsymbol{D}(\boldsymbol{k}_{||})_{-,\perp} - \boldsymbol{D}(\boldsymbol{k}_{||})_{-,||}. \tag{42}$$

We find that the + source function for the image source can be obtained from the – source function of the source by reversing the sign of the parallel component. With $\boldsymbol{D}(\boldsymbol{K}_{||})_{-}=\boldsymbol{D}(\boldsymbol{k}_{||})_{-,\perp}+\boldsymbol{D}(\boldsymbol{k}_{||})_{-,||}$, and $\boldsymbol{D}(\boldsymbol{K}_{||})_{-,||}=-\boldsymbol{e}_z\times[\boldsymbol{e}_z\times\boldsymbol{D}(\boldsymbol{k}_{||})_{-}]$, we can also write equation (42) as

$$\boldsymbol{D}(\boldsymbol{k}_{||})_{+}^{\text{im}} = \boldsymbol{D}(\boldsymbol{k}_{||})_{-} + 2\boldsymbol{e}_z\times[\boldsymbol{e}_z\times\boldsymbol{D}(\boldsymbol{k}_{||})_{-}]. \tag{43}$$

The reflected field is given by the second term on the right-hand side of equation (31), and with the identity (35) this can also be written as

$$\boldsymbol{B}(\boldsymbol{r})_{\text{r}} = \frac{i\omega\mu_{\text{o}}}{8\pi^2}\int \mathrm{d}^2\boldsymbol{k}_{||}\frac{1}{\beta}e^{i\boldsymbol{K}_{+}\cdot\boldsymbol{r}}\big\langle\boldsymbol{K}_{-}\times\boldsymbol{D}(\boldsymbol{k}_{||})_{-} - 2\boldsymbol{e}_z\big\{\boldsymbol{e}_z\cdot\big[\boldsymbol{K}_{-}\times\boldsymbol{D}(\boldsymbol{k}_{||})_{-}\big]\big\}\big\rangle. \tag{44}$$

For the field by the image source, equation (40), we need the cross-product of $\boldsymbol{D}(\boldsymbol{k}_{||})_{+}^{\text{im}}$ with $\boldsymbol{K}_{+}$. To this end we set $\boldsymbol{a}=\boldsymbol{e}_z\times\boldsymbol{D}(\boldsymbol{k}_{||})_{-}$ in equation (34). The second term on the right-hand side vanishes and with a vector identity the result can be written as

$$\boldsymbol{K}_{+}\times\{\boldsymbol{e}_z\times[\boldsymbol{e}_z\times\boldsymbol{D}(\boldsymbol{k}_{||})_{-}]\} = -\boldsymbol{e}_z\big\{\boldsymbol{e}_z\cdot\big[\boldsymbol{K}_{-}\times\boldsymbol{D}(\boldsymbol{k}_{||})_{-}\big]\big\} - \beta\boldsymbol{e}_z\times\boldsymbol{D}(\boldsymbol{k}_{||})_{-}. \tag{45}$$

Furthermore we have from equation (20)

$$\boldsymbol{K}_{+}\times\boldsymbol{D}(\boldsymbol{k}_{||})_{-}=\boldsymbol{K}_{-}\times\boldsymbol{D}(\boldsymbol{k}_{||})_{-}+2\beta\boldsymbol{e}_{z}\times\boldsymbol{D}(\boldsymbol{k}_{||})_{-}. \tag{46}$$

Then we cross the representation (43) of $\boldsymbol{D}(\boldsymbol{k}_{||})_{+}^{\mathrm{im}}$ with $\boldsymbol{K}_{+}$ and use equations (45) and (46). This yields

$$\boldsymbol{K}_{+}\times\boldsymbol{D}(\boldsymbol{k}_{||})_{+}^{\mathrm{im}}=\boldsymbol{K}_{-}\times\boldsymbol{D}(\boldsymbol{k}_{||})_{-}-2\boldsymbol{e}_{z}\{\boldsymbol{e}_{z}\cdot[\boldsymbol{K}_{-}\times\boldsymbol{D}(\boldsymbol{k}_{||})_{-}]\}, \tag{47}$$

and when we substitute the right-hand side in equation (40) for the field by the image source, the result is exactly equation (44) for the reflected field. This shows that the current density (38) and the source function (43) represent an image source that radiates a field in $z>0$ which is identical to the reflected field.

9. Localized source

The source functions $\boldsymbol{D}(\boldsymbol{k}_{\|})_{\pm}$ from equation (25) depend on the origin of coordinates. When a source is localized around some point $\boldsymbol{r}_{\mathrm{o}}$, as in figure 2, then it is more convenient to take this point as the origin of coordinates for the source functions. We make the change of variables $\boldsymbol{\xi}=\boldsymbol{r}-\boldsymbol{r}_{\mathrm{o}}$ in equation (25), which gives

$$\boldsymbol{D}(\boldsymbol{k}_{||})_{\pm}=e^{-i\boldsymbol{K}_{\pm}\cdot\boldsymbol{r}_{\mathrm{o}}}\boldsymbol{d}(\boldsymbol{k}_{||})_{\pm}, \tag{48}$$

with the new source functions $\boldsymbol{d}(\boldsymbol{k}_{\|})_{\pm}$ defined as

$$\boldsymbol{d}(\boldsymbol{k}_{||})_{\pm}=\frac{i}{\omega}\int\mathrm{d}^{3}\xi\boldsymbol{j}(\boldsymbol{\xi})e^{-i\boldsymbol{K}_{\pm}\cdot\boldsymbol{\xi}}. \tag{49}$$

The field emitted by the source, equation (26), then becomes

$$\boldsymbol{B}(\boldsymbol{r})=\frac{i\omega\mu_{\mathrm{o}}}{8\pi^{2}}\int\mathrm{d}^{2}\boldsymbol{k}_{||}\frac{1}{\beta}e^{i\boldsymbol{K}_{\pm}\cdot(\boldsymbol{r}-\boldsymbol{r}_{\mathrm{o}})}\boldsymbol{K}_{\pm}\times\boldsymbol{d}(\boldsymbol{k}_{||})_{\pm}. \tag{50}$$

The $\boldsymbol{r}$ dependence of the field enters through $\exp[i\boldsymbol{K}_{\pm}\cdot(\boldsymbol{r}-\boldsymbol{r}_{\mathrm{o}})]$, and it is now more obvious that the field emanates from the neighbourhood of the point $\boldsymbol{r}_{\mathrm{o}}$. The reflected field is given by the second term on the right-hand side of equation (31), and with $\boldsymbol{K}_{-}\cdot\boldsymbol{r}_{\mathrm{o}}=\boldsymbol{K}_{+}\cdot\boldsymbol{r}_{\mathrm{o}}^{\mathrm{im}}$ this becomes

$$\boldsymbol{B}(\boldsymbol{r})_{\mathrm{r}}=-\frac{i\omega\mu_{\mathrm{o}}}{8\pi^{2}}\int\mathrm{d}^{2}\boldsymbol{k}_{||}\frac{1}{\beta^{2}}e^{i\boldsymbol{K}_{+}\cdot(\boldsymbol{r}-\boldsymbol{r}_{\mathrm{o}}^{\mathrm{im}})}\boldsymbol{K}_{+}\times\{\boldsymbol{e}_{z}\times[\boldsymbol{K}_{-}\times\boldsymbol{d}(\boldsymbol{k}_{||})_{-}]\}. \tag{51}$$

The $\boldsymbol{r}$ dependence of the reflected field enters as $\exp[i\boldsymbol{K}_{+}\cdot(\boldsymbol{r}-\boldsymbol{r}_{\mathrm{o}}^{\mathrm{im}})]$, and it appears that this field is radiated by a localized source around the image position $\boldsymbol{r}_{\mathrm{o}}^{\mathrm{im}}$. Alternatively, the image source function from equation (43) is with equation (48) and $\boldsymbol{K}_{-}\cdot\boldsymbol{r}_{\mathrm{o}}=\boldsymbol{K}_{+}\cdot\boldsymbol{r}_{\mathrm{o}}^{\mathrm{im}}$:

$$\boldsymbol{D}(\boldsymbol{k}_{||})_{+}^{\mathrm{im}}=e^{-i\boldsymbol{K}_{+}\cdot\boldsymbol{r}_{\mathrm{o}}^{\mathrm{im}}}\boldsymbol{d}(\boldsymbol{k}_{||})_{+}^{\mathrm{im}}, \tag{52}$$

where

$$\boldsymbol{d}(\boldsymbol{k}_{||})_{+}^{\mathrm{im}} = \boldsymbol{d}(\boldsymbol{k}_{||})_{-} + 2\boldsymbol{e}_z \times \left[\boldsymbol{e}_z \times \boldsymbol{d}\left(\boldsymbol{k}_{||}\right)_{-}\right]. \tag{53}$$

With equation (40) we then have

$$\boldsymbol{B}(\boldsymbol{r})_{\mathrm{r}} = \frac{i\omega\mu_{\mathrm{o}}}{8\pi^2}\int \mathrm{d}^2\boldsymbol{k}_{||}\frac{1}{\beta}e^{i\boldsymbol{K}_{+}\cdot(\boldsymbol{r}-\boldsymbol{r}_{\mathrm{o}}^{\mathrm{im}})}\boldsymbol{K}_{+} \times \boldsymbol{d}\left(\boldsymbol{k}_{||}\right)_{+}^{\mathrm{im}}, \tag{54}$$

and this is the alternative to representation (51). We now note that equations (50) and (54) are identical in form. The source radiation emanates from the point $\boldsymbol{r}_{\mathrm{o}}$ and has $\boldsymbol{d}(\boldsymbol{k}_{||})_{\pm}$ as it source function. The reflected radiation, when viewed as the image source field, appears to come from $\boldsymbol{r}_{\mathrm{o}}^{\mathrm{im}}$, and it has $\boldsymbol{d}(\boldsymbol{k}_{||})_{+}^{\mathrm{im}}$ as its source function.

10. Dipoles

When the source is an atom or molecule in an excited state, the emitted radiation during spontaneous emission is usually electric dipole radiation. The time-dependent electric dipole moment of the oscillating electron cloud has the form

$$\boldsymbol{p}_e(t) = \mathrm{Re}(\boldsymbol{p}_e e^{-i\omega t}), \tag{55}$$

with $\boldsymbol{p}_e$ the complex amplitude, which is referred to as the electric dipole moment. When the atom or molecule is located at $\boldsymbol{r}_{\mathrm{o}}$, the corresponding current density is

$$\boldsymbol{j}(\boldsymbol{r}) = -i\omega\boldsymbol{p}_e\delta(\boldsymbol{r} - \boldsymbol{r}_{\mathrm{o}}), \tag{56}$$

and with equation (49) we find for the source functions

$$\boldsymbol{d}(\boldsymbol{k}_{||})_{\pm} = \boldsymbol{p}_e. \tag{57}$$

Apparently, for an electric dipole the + and − source functions are the same. For a point source, the region $a<z<b$ in figure 2 becomes $z=z_{\mathrm{o}}$, so the angular spectrum of the emitted dipole radiation is given by equation (50) with $\boldsymbol{d}(\boldsymbol{k}_{||})_{\pm}=\boldsymbol{p}_e$, and where the upper (lower) sign holds for $z>z_{\mathrm{o}}$ ($z<z_{\mathrm{o}}$). The reflected field is given by equation (51) with $\boldsymbol{d}(\boldsymbol{k}_{||})_{-}=\boldsymbol{p}_e$. Alternatively, in the image approach the reflected field is given by representation (54). For the image source function, the parallel component changes sign, so we have

$$\boldsymbol{d}(\boldsymbol{k}_{||})_{+}^{\mathrm{im}} = \boldsymbol{p}_{e,\perp} - \boldsymbol{p}_{e,||} \equiv \boldsymbol{p}_e^{\mathrm{im}}. \tag{58}$$

When compared to equation (57), we see that the image source is again an electric dipole $\boldsymbol{p}_e^{\mathrm{im}}$, but with its parallel component reversed in sign. This, of course, is a well-known fact and also follows easily from considerations as in figure 3.

Less trivial is the case of a magnetic dipole $\boldsymbol{p}_m$ at $\boldsymbol{r}_o$. The time-dependent magnetic dipole moment is given by equation (55) with $\boldsymbol{p}_e \to \boldsymbol{p}_m$. The complex amplitude of the current density is [25]

$$\boldsymbol{j}(\boldsymbol{r}) = -\boldsymbol{p}_m \times \nabla\delta(\boldsymbol{r} - \boldsymbol{r}_o). \tag{59}$$

From equation (49) we obtain for the source functions

$$\boldsymbol{d}(\boldsymbol{k}_{||})_\pm = \frac{1}{\omega}\boldsymbol{p}_m \times \boldsymbol{K}_\pm, \tag{60}$$

and here the + and − source functions are different. In order to find the image source we need to reverse the parallel part of $\boldsymbol{d}(\boldsymbol{k}_{||})_-$. To this end we first write $\boldsymbol{p}_m = \boldsymbol{p}_{m,\perp} + \boldsymbol{p}_{m,||}$ and use equation (20) for $\boldsymbol{K}_-$. This gives

$$\boldsymbol{d}(\boldsymbol{k}_{||})_- = \frac{1}{\omega}(\boldsymbol{p}_{m,\perp} \times \boldsymbol{k}_{||} - \beta\boldsymbol{p}_{m,||} \times \boldsymbol{e}_z + \boldsymbol{p}_{m,||} \times \boldsymbol{k}_{||}). \tag{61}$$

The first two terms on the right-hand side are parallel to the xy-plane and the third term is perpendicular to the xy-plane. Therefore the image source function is

$$\boldsymbol{d}(\boldsymbol{k}_{||})_+^{\text{im}} = \frac{1}{\omega}(-\boldsymbol{p}_{m,\perp} \times \boldsymbol{k}_{||} + \beta\boldsymbol{p}_{m,||} \times \boldsymbol{e}_z + \boldsymbol{p}_{m,||} \times \boldsymbol{k}_{||}), \tag{62}$$

which can also be written in a form similar to equation (60) as

$$\boldsymbol{d}(\boldsymbol{k}_{||})_+^{\text{im}} = \frac{1}{\omega}\boldsymbol{p}_m^{\text{im}} \times \boldsymbol{K}_+, \tag{63}$$

with

$$\boldsymbol{p}_m^{\text{im}} = \boldsymbol{p}_{m,||} - \boldsymbol{p}_{m,\perp}, \tag{64}$$

for the image dipole moment. So the mirror image of a magnetic dipole is again a magnetic dipole, but it has its perpendicular component reversed in sign, as compared to the original dipole.

11. Electric quadrupole

An electric quadrupole has a current density [25]

$$\boldsymbol{j}(\boldsymbol{r}) = \frac{i\omega}{6}\overleftrightarrow{\mathrm{Q}} \cdot \nabla\delta(\boldsymbol{r} - \boldsymbol{r}_o), \tag{65}$$

with $\overleftrightarrow{\mathrm{Q}}$ a symmetric Cartesian tensor. From equation (49) we find the source functions to be

$$\boldsymbol{d}(\boldsymbol{k}_{||})_\pm = \frac{-i}{6}\overleftrightarrow{\mathrm{Q}} \cdot \boldsymbol{K}_\pm. \tag{66}$$

We now want to find the image source function $\boldsymbol{d}(\boldsymbol{k}_{||})_{+}^{\mathrm{im}}$, and express it in the form

$$\boldsymbol{d}(\boldsymbol{k}_{||})_{+}^{\mathrm{im}} = \frac{-i}{6}\overleftrightarrow{\mathrm{Q}}^{\mathrm{im}} \cdot \boldsymbol{K}_{+}. \tag{67}$$

Then equations (66) and (67) are identical in form, and we can consider $\overleftrightarrow{Q}^{\mathrm{im}}$ the image of the electric quadrupole.

In order to obtain $\overleftrightarrow{Q}^{\mathrm{im}}$ we need to find the parallel and perpendicular components of the right-hand side of equation (66). It will turn out to be convenient to use spherical basis vectors $\boldsymbol{e}_{\mu}$, $\mu = -1, 0, 1$, rather than Cartesian basis vectors. This basis is defined as

$$\boldsymbol{e}_{\pm 1} = \frac{1}{\sqrt{2}}(\mp \boldsymbol{e}_x - i\boldsymbol{e}_y), \tag{68}$$

and $\boldsymbol{e}_0 = \boldsymbol{e}_z$. We then define a basis of unit tensors as [21]

$$\overleftrightarrow{\mathrm{E}}_q^{(k)} = \sum_{\mu\mu'} (1\ \mu\ 1\ \mu'|k\ q)\boldsymbol{e}_{\mu}\boldsymbol{e}_{\mu'}, \tag{69}$$

with $(1\ \mu\ 1\ \mu'|k\ q)$ Clebsch–Gordan coefficients. These coefficients are only nonzero for $k = 0, 1$ and 2, and for a given k the possible q values are $-k, \ldots, k$. So for $k = 0$ we only have $q = 0$, and for $k = 1$ we have $q = -1, 0, 1$ and for $k = 2$ there are five q values. This gives nine independent tensors, just like there are nine independent Cartesian tensors of the form $\boldsymbol{e}_{\alpha}\boldsymbol{e}_{\beta}$ with $\alpha, \beta = x, y$, or z. Then we expand $\overleftrightarrow{\mathrm{Q}}$ as

$$\overleftrightarrow{\mathrm{Q}} = \sum_{kq} Q_q^{(k)} \overleftrightarrow{\mathrm{E}}_q^{(k)*}. \tag{70}$$

The expansion coefficients $Q_q^{(k)}$ are complex numbers, and for a symmetric Cartesian tensor $\overleftrightarrow{\mathrm{Q}}$ the coefficients with $k = 1$ vanish. We then obtain the expression

$$\boldsymbol{d}(\boldsymbol{k}_{||})_{-} = \frac{-i}{6}\sum_{\mu} \boldsymbol{e}_{\mu}^{*} \sum_{kq\mu'} Q_q^{(k)} (1\ \mu\ 1\ \mu'|k\ q)(\boldsymbol{e}_{\mu'}^{*} \cdot \boldsymbol{K}_{-}), \tag{71}$$

for this source function, where we have used that the Clebsch–Gordan coefficients are real. The terms with $\mu = \pm 1$ are in the xy-plane and the term with $\mu = 0$ is along the z-axis. For the source function of the image source we therefore get a factor $(-1)^{\mu}$ in equation (71), with everything else the same. Then we have

$$\boldsymbol{e}_{\mu'}^{*} \cdot \boldsymbol{K}_{-} = (-1)^{\mu'+1}\boldsymbol{e}_{\mu'}^{*} \cdot \boldsymbol{K}_{+}, \tag{72}$$

which gives a factor of $(-1)^{\mu+\mu'+1}$. The Clebsch–Gordan coefficients are only nonzero for $\mu + \mu' = q$, so we can replace this with $(-1)^{q+1}$. We then obtain

$$\boldsymbol{d}(\boldsymbol{k}_{||})_{+}^{\mathrm{im}} = \frac{-i}{6}\sum_{kq\mu\mu'} (-1)^{q+1} Q_q^{(k)} (1\ \mu\ 1\ \mu'|k\ q)\boldsymbol{e}_{\mu}^{*}(\boldsymbol{e}_{\mu'}^{*} \cdot \boldsymbol{K}_{+}), \tag{73}$$

which is with equation (69)

$$\boldsymbol{d}(\boldsymbol{k}_{||})_{+}^{\mathrm{im}} = \frac{-i}{6}\sum_{kq}(-1)^{q+1}Q_q^{(k)}(\overleftrightarrow{\mathrm{E}}_q^{(k)*}\cdot\boldsymbol{K}_{+}). \tag{74}$$

Comparison with equation (67) then shows that the image quadrupole tensor is

$$\overleftrightarrow{\mathrm{Q}}^{\mathrm{im}} = \sum_{kq}(-1)^{q+1}Q_q^{(k)}\overleftrightarrow{\mathrm{E}}_q^{(k)*}, \tag{75}$$

and comparison with equation (70) shows that the component $Q_q^{(k)}$ picks up a factor $(-1)^{q+1}$ upon reflection.

12. Angular distribution of the emitted power

When a small source is located near the surface, then the emitted radiation can be observed by a detector in the far field. This radiation is the sum of the field emitted by the source, represented by equation (26) with the upper sign, and the field of the image source, given by equation (40). The field in the region $z>b$ is

$$\boldsymbol{B}(\boldsymbol{r}) = \frac{i\omega\mu_o}{8\pi^2}\int \mathrm{d}^2\boldsymbol{k}_{||}\frac{1}{\beta}e^{i\boldsymbol{K}_{+}\cdot\boldsymbol{r}}\boldsymbol{K}_{+}\times\left[\boldsymbol{D}(\boldsymbol{k}_{||})_{+} + \boldsymbol{D}(\boldsymbol{k}_{||})_{+}^{\mathrm{im}}\right], \tag{76}$$

and the corresponding electric field follows from $\boldsymbol{E}(\boldsymbol{r}) = (ic^2/\omega)\nabla\times\boldsymbol{B}(\boldsymbol{r})$, which is

$$\boldsymbol{E}(\boldsymbol{r}) = -\frac{i}{8\pi^2\varepsilon_o}\int \mathrm{d}^2\boldsymbol{k}_{||}\frac{1}{\beta}e^{i\boldsymbol{K}_{+}\cdot\boldsymbol{r}}\boldsymbol{K}_{+}\times\left\{\boldsymbol{K}_{+}\times\left[\boldsymbol{D}(\boldsymbol{k}_{||})_{+} + \boldsymbol{D}(\boldsymbol{k}_{||})_{+}^{\mathrm{im}}\right]\right\}. \tag{77}$$

For a detector in the far field, located at field point $\boldsymbol{r}$, we need to consider the solution for $\boldsymbol{r}$ large. An asymptotic expansion of an angular spectrum can be made with the method of stationary phase [26], and the result takes the general form

$$\int \mathrm{d}^2\boldsymbol{k}_{||}\frac{1}{\beta}e^{i\boldsymbol{K}_{+}\cdot\boldsymbol{r}}W(\boldsymbol{k}_{||}) \approx -2\pi i\frac{e^{ik_o r}}{r}W(\boldsymbol{k}_{||,o}), \tag{78}$$

where $W(\boldsymbol{k}_{||})$ is any function of $\boldsymbol{k}_{||}$. At the right-hand side this function is evaluated at the stationary point $\boldsymbol{k}_{||,o}$ in the $\boldsymbol{k}_{||}$ plane. With (r,θ,ϕ) the spherical coordinates of the location $\boldsymbol{r}$ of the detector, this stationary point is

$$\boldsymbol{k}_{||,o} = k_o\sin\theta(\boldsymbol{e}_x\cos\phi + \boldsymbol{e}_y\sin\phi). \tag{79}$$

The value of $\boldsymbol{K}_{+}$ in the stationary point is found to be $k_o\hat{\boldsymbol{r}}$, with $\hat{\boldsymbol{r}}$ the unit vector in the observation direction. We obtain for the asymptotic approximation of the magnetic field

$$\boldsymbol{B}(\boldsymbol{r}) \approx \frac{\omega\mu_o k_o}{4\pi r} e^{ik_o r}\hat{\boldsymbol{r}} \times \left[\boldsymbol{D}\left(\boldsymbol{k}_{||,o}\right)_+ + \boldsymbol{D}\left(\boldsymbol{k}_{||,o}\right)_+^{\mathrm{im}}\right]. \tag{80}$$

For the electric field a similar expression is found, and the result is related to the magnetic field by

$$\boldsymbol{E}(\boldsymbol{r}) \approx -c\hat{\boldsymbol{r}} \times \boldsymbol{B}(\boldsymbol{r}). \tag{81}$$

The Poynting vector representing the energy flow in an electromagnetic field is in general defined as

$$\boldsymbol{S}(\boldsymbol{r}) = \frac{1}{2\mu_o}\mathrm{Re}\boldsymbol{E}(\boldsymbol{r}) \times \boldsymbol{B}(\boldsymbol{r})^*. \tag{82}$$

Here, $\boldsymbol{E}(\boldsymbol{r})$ and $\boldsymbol{B}(\boldsymbol{r})$ are the complex amplitudes of the electric and magnetic fields, respectively, but $\boldsymbol{S}(\boldsymbol{r})$ is in the time domain. We have dropped terms which oscillate with twice the optical frequency, which has the consequence that $\boldsymbol{S}(\boldsymbol{r})$ is independent of time. With equation (81), a vector identity and $\hat{\boldsymbol{r}} \cdot \boldsymbol{B}(\boldsymbol{r})^* = 0$ this becomes

$$\boldsymbol{S}(\boldsymbol{r}) = \frac{c}{2\mu_o}[\boldsymbol{B}(\boldsymbol{r}) \cdot \boldsymbol{B}(\boldsymbol{r})^*]\hat{\boldsymbol{r}}. \tag{83}$$

We write an equal sign instead of $\approx$, which we shall do from here on. The detected power per unit solid angle into the direction $\hat{\boldsymbol{r}}$ is

$$\frac{\mathrm{d}P}{\mathrm{d}\Omega} = r^2\boldsymbol{S}(\boldsymbol{r}) \cdot \hat{\boldsymbol{r}}, \tag{84}$$

and with equations (80) and (83) this becomes

$$\begin{aligned}\frac{\mathrm{d}P}{\mathrm{d}\Omega} = \frac{\omega^4}{32\pi^2\varepsilon_o c^3}\Big\langle&\left[\boldsymbol{D}\left(\boldsymbol{k}_{||,o}\right)_+ + \boldsymbol{D}\left(\boldsymbol{k}_{||,o}\right)_+^{\mathrm{im}}\right] \cdot \left[\boldsymbol{D}\left(\boldsymbol{k}_{||,o}\right)_+ + \boldsymbol{D}\left(\boldsymbol{k}_{||,o}\right)_+^{\mathrm{im}}\right]^* \\ &-\left\{\hat{\boldsymbol{r}} \cdot \left[\boldsymbol{D}\left(\boldsymbol{k}_{||,o}\right)_+ + \boldsymbol{D}\left(\boldsymbol{k}_{||,o}\right)_+^{\mathrm{im}}\right]\right\}\left\{\hat{\boldsymbol{r}} \cdot \left[\boldsymbol{D}\left(\boldsymbol{k}_{||,o}\right)_+ + \boldsymbol{D}\left(\boldsymbol{k}_{||,o}\right)_+^{\mathrm{im}}\right]^*\right\}\Big\rangle.\end{aligned} \tag{85}$$

The result (85) can be simplified further by using equations (48) and (52) for a localized source. In the stationary point we have $\boldsymbol{K}_{+,o} = k_o\hat{\boldsymbol{r}}$, so that

$$\boldsymbol{D}(\boldsymbol{k}_{||,o})_+ + \boldsymbol{D}(\boldsymbol{k}_{||,o})_+^{\mathrm{im}} = \left[\boldsymbol{d}(\boldsymbol{k}_{||,o})_+ + e^{-ik_o\hat{\boldsymbol{r}}\cdot(\boldsymbol{r}_o^{im}-\boldsymbol{r}_o)}\boldsymbol{d}(\boldsymbol{k}_{||,o})_+^{\mathrm{im}}\right]e^{-ik_o\hat{\boldsymbol{r}}\cdot\boldsymbol{r}_o}. \tag{86}$$

Since this expression is multiplied by its complex conjugate in equation (85), the phase factor $\exp(-ik_o\hat{\boldsymbol{r}} \cdot \boldsymbol{r}_o)$ cancels. Furthermore, with $\boldsymbol{r}_o^{\mathrm{im}} - \boldsymbol{r}_o = -2z_o\boldsymbol{e}_z$, and with $h = k_o z_o$ as the dimensionless distance between the source and the surface we have $-ik_o\hat{\boldsymbol{r}} \cdot (\boldsymbol{r}_o^{\mathrm{im}} - \boldsymbol{r}_o) = 2ih\cos\theta$. We then introduce the function

$$\mathscr{D}(\theta,\phi) = \boldsymbol{d}(\boldsymbol{k}_{||,o})_+ + e^{2ih\cos\theta}\boldsymbol{d}(\boldsymbol{k}_{||,o})_+^{\mathrm{im}}, \tag{87}$$

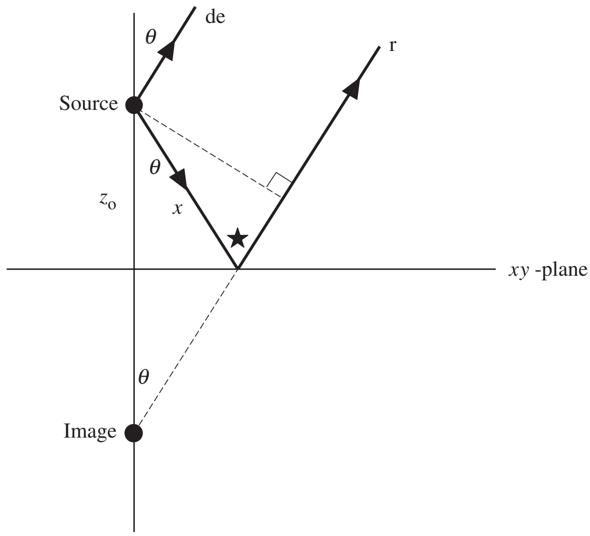

Figure 4. The directly emitted (de) wave travels directly from the source to the detector, and is observed under polar angle θ with the normal to the surface. The reflected (r) wave is also detected under angle θ, and since the angle of incidence equals the angle of reflection, the angle indicated by $\star$ is equal to 2θ. The wave emanating from the source as the incident wave is incident on the surface at angle θ, and therefore it leaves the source at angle θ with the normal. The normal distance between the source and the surface is z_o, and the distance travelled by the incident wave is indicated by x. We see from the figure that $x = z_o/\cos\theta$, and the additional distance travelled by the r-wave, as compared to the de-wave, is $x + x\cos 2\theta$, which is equal to $2z_o\cos\theta$. This explains the phase factor $\exp[ik_o(2z_o\cos\theta)]$ in equation (87) for the reflected wave. Since the reflected wave makes an angle θ with the normal to the surface, it intersects the normal line through the source at an angle θ. Therefore, the reflected wave appears to come from the location of the image source, a distance z_o below the surface.

which depends on θ and ϕ through the dependence of the source functions on $\boldsymbol{k}_{\|,o}$, with $\boldsymbol{k}_{\|,o}$ given by equation (79). The first term on the right-hand side of equation (87) comes from the radiation that is emitted by the source directly towards the detector, and the second term represents radiation that is emitted towards the surface, and then travels towards the detector after reflection. The factor $\exp(2ih\cos\theta)$ accounts for the retardation between the two signals, as is illustrated in figure 4. The figure also shows that the reflected radiation appears to come from the image source. The final result for the detected power per unit solid angle then takes the elegant form

$$\frac{\mathrm{d}P}{\mathrm{d}\Omega} = \frac{\omega^4}{32\pi^2\varepsilon_o c^3}\left\{\mathscr{D}(\theta,\phi)\cdot\mathscr{D}(\theta,\phi)^* - \left[\hat{\boldsymbol{r}}\cdot\mathscr{D}(\theta,\phi)\right]\left[\hat{\boldsymbol{r}}\cdot\mathscr{D}(\theta,\phi)^*\right]\right\}, \quad (88)$$

which holds for any localized source near the surface. The angle dependence enters through the functions $\mathscr{D}(\theta, \phi)$, and through the angle dependence of the unit vector $\hat{\boldsymbol{r}}$.

For an electric dipole with dipole moment $\boldsymbol{p}_e$ the functions $\boldsymbol{d}(\boldsymbol{k}_{\|})_+$ of the source and $\boldsymbol{d}(\boldsymbol{k}_{\|})_+^{\text{im}}$ of the image are independent of $\boldsymbol{k}_{\|}$, according to equations (57) and (58), so we find immediately

$$\mathscr{D}(\theta,\phi) = \boldsymbol{p}_e + e^{2ih\cos\theta}\boldsymbol{p}_e^{\text{im}}. \tag{89}$$

For a magnetic dipole these functions do depend on $\boldsymbol{k}_{\|}$, as follows from equations (60) and (63). They depend on $\boldsymbol{k}_{\|}$ through $\boldsymbol{K}_+$, which becomes $k_{\text{o}}\hat{\boldsymbol{r}}$ in the stationary point, and we find

$$\mathscr{D}(\theta,\phi) = -\frac{1}{c}\hat{\boldsymbol{r}} \times (\boldsymbol{p}_m + e^{2ih\cos\theta}\boldsymbol{p}_m^{\text{im}}). \tag{90}$$

Similarly, we obtain from equations (66) and (67)

$$\mathscr{D}(\theta,\phi) = -\frac{ik_{\text{o}}}{6}\hat{\boldsymbol{r}} \cdot (\overleftrightarrow{Q} + e^{2ih\cos\theta}\overleftrightarrow{Q}^{\text{im}}), \tag{91}$$

for an electric quadrupole. Equations (89)–(91) clearly show that the field by the image source is accounted for by adding the image dipole moment (image quadrupole tensor) to the dipole moment (quadrupole tensor). For the image term the retardation factor $\exp(2ih\cos\theta)$ appears, as a result of the fact that this radiation first travels to the surface where it subsequently reflects, as illustrated in figure 4. It is interesting to see that in this asymptotic limit the explicit dependence on the location $\boldsymbol{r}_{\text{o}}$ of the multipole drops out.

13. Power distribution of an electric quadrupole

As a nontrivial example we consider an electric quadrupole located near the surface. Let the quadrupole tensor be given by

$$\overleftrightarrow{\mathrm{Q}} = Q\overleftrightarrow{\mathrm{E}}_q^{(2)^*}, \tag{92}$$

with Q a complex number and $q = -2, \ldots, 2$. From equations (70) and (75) we find that the image quadrupole moment is given by

$$\overleftrightarrow{\mathrm{Q}}^{\text{im}} = Q(-1)^{q+1}\overleftrightarrow{\mathrm{E}}_q^{(2)^*} = (-1)^{q+1}\overleftrightarrow{\mathrm{Q}}. \tag{93}$$

The function $\mathscr{D}(\theta, \phi)$ becomes

$$\mathscr{D}(\theta,\phi) = -\frac{ik_{\text{o}}}{6}\hat{\boldsymbol{r}} \cdot \overleftrightarrow{Q}[1 + (-1)^{q+1}e^{2ih\cos\theta}], \tag{94}$$

and from equation (88) we find for the emitted power per unit solid angle

$$\frac{\mathrm{d}P}{d\Omega} = |Q|^2\frac{ck_{\text{o}}^6}{1152\pi^2\varepsilon_{\text{o}}}|1 + (-1)^{q+1}e^{2ih\cos\theta}|^2\left[\left(\hat{\boldsymbol{r}} \cdot \overleftrightarrow{\mathrm{E}}_q^{(2)}\right) \cdot \left(\hat{\boldsymbol{r}} \cdot \overleftrightarrow{\mathrm{E}}_q^{(2)^*}\right) - |\hat{\boldsymbol{r}} \cdot \overleftrightarrow{\mathrm{E}}_q^{(2)} \cdot \hat{\boldsymbol{r}}|^2\right]. \tag{95}$$

As an illustration, let $q=0$. From equation (69) and the tabulated values of the Clebsch–Gordan coefficients we find explicitly

$$\overleftrightarrow{\mathrm{E}}_0^{(2)} = \frac{1}{\sqrt{6}}(\boldsymbol{e}_1\boldsymbol{e}_{-1} + 2\boldsymbol{e}_0\boldsymbol{e}_0 + \boldsymbol{e}_{-1}\boldsymbol{e}_1), \tag{96}$$

in spherical unit vectors, and with equation (68) this becomes

$$\overleftrightarrow{\mathrm{E}}_0^{(2)} = \frac{1}{\sqrt{6}}(2\boldsymbol{e}_z\boldsymbol{e}_z - 2\boldsymbol{e}_x\boldsymbol{e}_x - \boldsymbol{e}_y\boldsymbol{e}_y), \tag{97}$$

in Cartesian unit vectors. We furthermore have $\hat{\boldsymbol{r}} = \sin\theta(\boldsymbol{e}_x\cos\phi + \boldsymbol{e}_y\sin\phi) + \boldsymbol{e}_z\cos\theta$, which gives

$$\left(\hat{\boldsymbol{r}}\cdot\overleftrightarrow{\mathrm{E}}_0^{(2)}\right)\cdot\left(\hat{\boldsymbol{r}}\cdot\overleftrightarrow{\mathrm{E}}_0^{(2)*}\right) - |\hat{\boldsymbol{r}}\cdot\overleftrightarrow{\mathrm{E}}_0^{(2)}\cdot\hat{\boldsymbol{r}}|^2 = \frac{3}{8}(\sin 2\theta)^2, \tag{98}$$

and this yields for the angular distribution of the emitted power

$$\frac{\mathrm{d}P}{\mathrm{d}\Omega} = |Q|^2\frac{ck_\mathrm{o}^6}{1536\pi^2\varepsilon_\mathrm{o}}[1-\cos(2h\cos\theta)](\sin 2\theta)^2. \tag{99}$$

This intensity pattern is shown in figure 5, together with the angular distribution of the radiation for this quadrupole in free space. The lobe structure is a result of interference between the radiation emitted directly by the source towards the detector and the radiation that is first reflected off the mirror.

14. Conclusions

When electromagnetic radiation is incident upon a perfectly-conducting medium, a current density is induced on the surface of the material. This surface current density $\boldsymbol{i}(\boldsymbol{r})$ is the solution of the integral equation (8), which has the incident magnetic field as inhomogeneous term. We have shown that for a flat surface of infinite extent the solution of this equation can be obtained in closed form, and is given by equation (11). This remarkably simple expression shows that the current density at the point $\boldsymbol{r}$ of the surface is determined by the value of the incident magnetic field at that same point. With the current density known, the total magnetic field is the sum of the incident field and the field generated by $\boldsymbol{i}(\boldsymbol{r})$, and the result is given by equation (15), involving the Green's function for the scalar free-space Helmholtz equation. The total magnetic field is determined entirely by the value of the incident magnetic field at the surface of the conductor, irrespective the source of the radiation. With Weyl's representation (16) of the Green's function, the magnetic field in $z>0$ can also be written as an angular spectrum, as given by equation (21). The angular spectrum representation of the field in $z<0$ is given by equation (22), and it was shown that this field vanishes. The current density

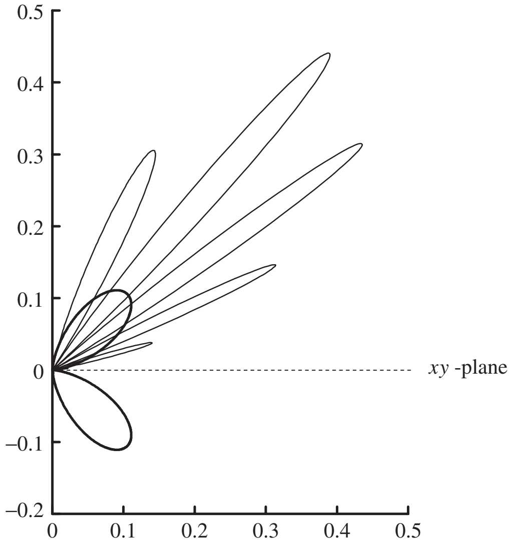

Figure 5. The thin line is a polar diagram of the angular intensity distribution of the radiation emitted by a $q = 0$ electric quadrupole near the surface of a perfect conductor, and the thick line is the emission pattern for the same quadrupole in free space. The radiation pattern is rotation symmetric around the normal to the surface. The dimensionless normal distance between the quadrupole and the surface is $h = 6\pi$, and both curves are scaled with the same overall constant.

generates a magnetic field, and inside the material this field cancels exactly the incident field.

When the incident field is radiation from a source with known current density $\mathbf{j}(\mathbf{r})$, the reflected field can be expressed in terms of a source function $\mathbf{D}(\mathbf{k}_\parallel)_-$, defined by equation (25), and the result is given by the second term on the right-hand side of equation (31). In section 8 we constructed an image source, and equation (42) or (43) shows that the source function of the image source can be obtained in a simple way from the source function of the source itself. It was shown explicitly that the image source radiates an electromagnetic field in $z > 0$ which is identical to the reflected field.

When the source is localized around some point $\mathbf{r}_o$ it is advantageous, but not necessary, to make the change of coordinates shown in figure 2. The dependence on the location $\mathbf{r}_o$ of the source then factors out as in equation (48), leading to the new source functions $\mathbf{d}(\mathbf{k}_\parallel)_\pm$. We illustrated in sections 10 and 11 that the approach with the source functions leads to a simple method for obtaining the mirror image of multipoles with, for example, the mirror image of an electric quadrupole given by equation (75). For a localized source, the power emitted per unit solid angle, including the directly emitted radiation by the source and the reflected field, was found to be given by equation (88). This expression holds for any source of radiation near the surface, and its evaluation only involves the source functions $\mathbf{d}(\mathbf{k}_\parallel)_\pm$.

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