Transmission of dipole radiation through interfaces and the phenomenon of anti-critical angles

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Radiation emitted by an electric dipole consists of traveling and evanescent plane waves. Usually, only the traveling waves are observable by a measurement in the far field, since the evanescent waves die out over a length of approximately a wavelength from the source. We show that when the radiation is passed through an interface with a medium with an index of refraction larger than the index of refraction of the embedding medium of the dipole, a portion of the evanescent waves are converted into traveling waves, and they become observable in the far field. The same conclusion holds when the waves pass through a layer of finite thickness. Waves that are transmitted under an angle larger than the so-called anti-critical angle \( \theta_{\text{a}}^{(1)} \) are shown to originate in evanescent dipole waves. In this fashion, part of the evanescent spectrum of the radiation becomes amenable to observation in the far field. We also show that in many situations the power in the far field coming from evanescent waves greatly exceeds the power originating in traveling waves. © 2004 Optical Society of America

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1. INTRODUCTION

When a monochromatic plane wave, traveling in a medium with index of refraction \( n_1 \), is incident on a planar interface with a medium of index of refraction \( n_2 < n_1 \), then the transmitted wave will be evanescent (exponentially decaying away from the surface) when the angle of incidence exceeds the critical angle \( \theta_\text{c} \). For later reference we indicate this angle by \( \theta_\text{c}^{(2)} \), and it is given by

\[
\sin \theta_\text{c}^{(2)} = n_2/n_1.
\]  

(1)

If this transmitted wave encounters a second interface parallel to the first, with a medium of index of refraction \( n_3 \), then the wave can emerge as either traveling or evanescent. For instance, if \( n_3 > n_1 \), then the wave transmitted through the layer will be traveling again.

Another example of the occurrence of a critical angle is when we consider only the waves in medium \( n_1 \) and medium \( n_2 \) but with a layer of medium \( n_2 \) in between. If \( n_3 < n_1 \), then there exists another critical angle \( \theta_\text{c}^{(1)} \):

\[
\sin \theta_\text{c}^{(1)} = n_3/n_1.
\]  

(2)

When the angle of incidence of the wave in medium \( n_1 \) exceeds the critical angle \( \theta_\text{c}^{(1)} \), then the emerging wave in medium \( n_3 \) is evanescent, no matter the character of the wave in the layer in between. In any case, upon transmission through an interface or a layer, a traveling wave can be converted into an evanescent wave, and vice versa.

Usually, an incident wave is traveling. However, in the established research on near-field optics it has become evident that evanescent waves play a crucial role.\(^1\)-\(^7\) In a scanning near-field microscope a probe, consisting of a fiber tip, is moved over a nanosized sample. The probe can either emit light for illumination of the sample (emission mode) or collect light scattered by the sample (collection mode). In the emission mode, the light coming out of the fiber tip contains evanescent waves that are produced by diffraction through the small aperture (the opening of the tip).\(^8\)-\(^10\) and in the collection mode, the modes of the fiber couple to the evanescent modes of the scattered light. In a different configuration, a sample can be illuminated by either a traveling or an evanescent wave, generated by total internal reflection, and the scattered field or the fluorescence (in case of a molecule) is observed by a macroscopic detector in the far field. In this case, the sample is positioned on an oil-immersion hemispherical lens. Traveling waves from the field scattered by the sample exit the lens under an angle of at most the critical angle for total internal reflection, given by Eq. (1), and this radiation is sometimes called “allowed light.” It has long been recognized,\(^11\)-\(^15\) however, that if one seeks to improve the resolution of such an imaging device into the nanometer region, then also the waves that appear under an angle larger than the critical angle have to be observed. Since this radiation has its origin in scattered evanescent waves, this radiation is given the name “forbidden light.”\(^16\)

Of particular interest is the radiation emitted by an electric dipole located in the vicinity of a surface, since such radiation is emitted by atoms and molecules as fluorescence. This radiation is a superposition of traveling and evanescent waves. Normally, the traveling waves are detected in the far field with a macroscopic device such as a photomultiplier, but since evanescent waves can be transformed into traveling waves at an interface, it should be possible to detect traveling waves in the far field that have their origin in evanescent dipole waves.\(^17\) These waves appear as forbidden light under an angle
larger than the critical angle for total internal reflection, and this radiation has been indeed observed for dipole radiation.\cite{15}

We shall consider the situation shown in Fig. 1, where the dipole \( \mathbf{d} \) is located on the \( z \) axis at a distance \( H \) above the first interface, which is the \( xy \) plane. The second boundary is at \( z = -L \), and the indices of refraction are \( n_1, n_2, \) and \( n_3 \), assumed to be positive. The dipole, embedded in medium \( n_1 \), oscillates harmonically as \( \mathbf{d}(t) = \text{Re}[\mathbf{d}(t) \exp(-i\omega t)] \). Without boundaries, the complex amplitude \( \mathbf{E}_s(r) \) of the dipole (source) field \( \mathbf{E}_s(r, t) = \text{Re}[\mathbf{E}_s(r) \exp(-i\omega t)] \) can be represented as an angular spectrum, according to\cite{19-21}

\[
\mathbf{E}_s(r) = \frac{i}{8\pi^2\varepsilon_0 n_1} \int \mathrm{d}^2k \frac{1}{\beta} [k^2 n_1^2 \mathbf{d} - (\mathbf{d} \cdot \mathbf{k}) \mathbf{k}] \\
\times \exp[i \mathbf{k} \cdot (r - H \mathbf{e}_z)], \quad z \neq H,
\]

with \( k_o = \omega/c \). This is a superposition of plane waves with wave vectors \( \mathbf{k} = \mathbf{k}_i + \beta \text{sgn}(z - H) \mathbf{e}_z \), and here \( \mathbf{k}_i \) indicates the component of \( \mathbf{k} \) parallel to the \( xy \) plane. The integration then runs over the entire \( \mathbf{k}_i \) plane. The parameter \( \beta \), which is the \( z \) component of \( \mathbf{k} \) for \( z > H \) and the negative of the \( z \) component of \( \mathbf{k} \) for \( z < H \), is defined as

\[
\beta = \begin{cases} 
\frac{\sqrt{k_o^2 - k^2}}{k_o} & \text{for } k_1 < k_o n_1 \\
\frac{i \sqrt{k_o^2 - k^2}}{k_o} & \text{for } k_1 > k_o n_1.
\end{cases}
\]

For \( k_1 < k_o n_1 \), \( \beta \) is real, and the corresponding wave is a traveling wave. On the other hand, when \( k_1 > k_o n_1 \), \( \beta \) is positive imaginary, representing an evanescent wave that decays exponentially away from the plane \( z = H \) on both sides. The corresponding magnetic field is given by

\[
\mathbf{B}_s(r) = -\frac{i}{\omega} \nabla \times \mathbf{E}_s(r),
\]

and it can be verified by inspection that the individual waves satisfy the source-free Maxwell equations for \( z > H \) and \( z < H \). These partial waves then serve as the incident field on the boundary with the \( xy \) plane, giving rise to reflection and transmission.

**Fig. 1.** An electric dipole, with dipole moment \( \mathbf{d} \), in a medium with index of refraction \( n_1 \), located a distance \( H \) above the \( xy \) plane and on the \( z \) axis. The region \( -L < z < 0 \) contains a dielectric material with index of refraction \( n_2 \), and the region \( z < -L \) is filled with a material with index of refraction \( n_3 \).

### 2. Wave Vectors and Polarization Convention

For a given incident plane wave with wave vector \( \mathbf{k} \), the resulting reflected and transmitted waves are again plane waves. Figure 2 shows the various wave vectors, including the decay direction in the case that the wave is evanescent. Boundary conditions at \( z = 0 \) and \( z = -L \) require that all waves have the same parallel component \( \mathbf{k}_i \). It will be convenient to introduce a dimensionless variable for the magnitude of \( \mathbf{k} \) by

\[
\alpha = k_i/k_o.
\]

If the incident wave is traveling, with an angle of incidence \( \theta_{inc} \), we have \( \alpha = n_1 \sin \theta_{inc} \). For a wave with wave vector \( \mathbf{k}_i (\alpha = r \text{ for the reflected wave, } t \text{ for transmitted, etc.}) \) in medium \( n_1 \), the wave number is \( k_o = k_o n_1 \), and since the parallel part of the wave vector is determined by the incident field, we have for the \( z \) component \( k_{a,z} = \pm k_o (n_1^2 - \alpha^2)^{1/2} \), leaving only the sign to be determined. We furthermore introduce the abbreviation

\[
v_i = \sqrt{n_i^2 - \alpha^2}, \quad i = 1, 2, 3,
\]

and it is understood that we take \( v_i \) to be positive imaginary when \( \alpha > n_i \). Parameter \( \beta \) from Eq. (4) then becomes \( \beta = k_o v_1 \), so that a wave vector of the source field is

\[
\mathbf{k} = \begin{cases} 
\mathbf{k}_i + k_o v_1 \mathbf{e}_z, & z > H \\
\mathbf{k}_i - k_o v_1 \mathbf{e}_z, & z < H.
\end{cases}
\]

The specular (reflected) wave has a wave vector

\[
\mathbf{k}_r = \mathbf{k}_i + k_o v_1 \mathbf{e}_z.
\]

corresponding to a wave traveling in the positive \( z \) direction (\( \alpha < n_1 \)) or decaying away from the \( xy \) plane (\( \alpha > n_1 \)). For the two waves in the layer, we have \( \mathbf{k}_z = \mathbf{k}_i \pm k_o v_2 \mathbf{e}_z \) corresponding to waves that travel or decay, as shown in Fig. 2. The transmitted wave travels or decays in the negative \( z \) direction, and therefore its wave vector is

\[
\mathbf{k}_t = \mathbf{k}_i - k_o v_2 \mathbf{e}_z.
\]

The most convenient way to calculate the reflected and transmitted fields is by first decomposing the partial waves of the source field into \( s \)- and \( p \)-polarized waves.\cite{22,23} Given \( \mathbf{k}_i \), we define the unit vector for \( s \) polarization by

\[
\mathbf{e}_s = \frac{1}{k_i} \mathbf{k}_i \times \mathbf{e}_z,
\]

which is the same for all waves. For \( p \) polarization we take

\[
\mathbf{e}_{p,s} = \frac{1}{k_o} \mathbf{k}_o \times \mathbf{e}_z,
\]

which depends on the corresponding wave vector \( \mathbf{k}_o \). For instance, for the \( t \) wave we have

\[
\mathbf{e}_{p,t} = -\frac{1}{n_3} [v_3(k_i/k_3) + \alpha \mathbf{e}_z].
\]
wave, represented by wave vector $k$. Here we have set $h \equiv k o H$. We then have $s \equiv n \sigma s, p$. (15) and (17), and the field in the layer $z > H$ is the sum of Eqs. (16) and (17). We write this as

$$E_r = \frac{ik_o}{8 \pi^2 \epsilon_o} \int d^2 k \sum_{\sigma, s, p} \frac{\exp(i \nu_1 h)}{\nu_1} (\mathbf{d} \cdot \mathbf{e}_{\sigma, r}) (\mathbf{d} \cdot \mathbf{e}_{\sigma}) e_{\sigma, r}, \quad z > H. \quad (16)$$

Each incident partial wave in Eq. (15) couples to a set of waves as shown in Fig. 2. The amplitudes of the remaining waves can be expressed in terms of Fresnel coefficients that determine the amplitude of each wave with respect to the amplitude of the corresponding partial incident wave. For the reflected wave we then obtain

$$E_r = \frac{ik_o}{8 \pi^2 \epsilon_o} \int d^2 k \sum_{\sigma, s, p} \frac{\exp(i \nu_1 h)}{\nu_1} R_{s, r}(a) \times (\mathbf{d} \cdot \mathbf{e}_{\sigma, r}) \exp(i \mathbf{k} \cdot \mathbf{r}), \quad z > 0, \quad (17)$$

with $R_{s, r}(a)$ the Fresnel reflection coefficients for $s$-polarized and $p$-polarized waves. The Fresnel coefficients can be obtained in the usual way by applying the boundary conditions at $z = 0$ and $z = -L$ for a given incident partial wave. For reference, we have listed these in Appendix A for the configuration shown in Fig. 2. The total field in $z > H$ is then the sum of Eqs. (16) and (17). We write this as

$$E_r = \frac{ik_o}{8 \pi^2 \epsilon_o} \int d^2 k \sum_{\sigma, s, p} \frac{\exp(i \nu_1 h)}{\nu_1} \left[ \exp(-i \nu_1 h) \mathbf{d} \cdot \mathbf{e}_{\sigma, r} + \exp(i \nu_1 h) R_{s, r}(a) \mathbf{d} \cdot \mathbf{e}_{\sigma} \right], \quad z > H. \quad (18)$$

The field in the region $0 < z < H$ is the sum of Eqs. (15) and (17), and the field in the layer $-L < z < 0$ can be constructed by using the appropriate Fresnel coefficients. We shall omit the explicit expressions here. The transmitted field in the region $z < -L$ can be expressed as
with $T_{\phi}(\alpha)$ the Fresnel transmission coefficients.

4. ASYMPTOTIC APPROXIMATION

The radiation is detected at a field point with spherical coordinates $(r, \theta, \phi)$ and with $r$ large (far field). From Eqs. (18) and (19) we can derive the far-field approximations for the fields in $z > H$ and $z < -L$, respectively, with the method of stationary phase.\textsuperscript{24} 26 Both expressions are integrals over $k_0$. In the method of stationary phase it is asserted that the major contribution for $r$ large and $\theta$ and $\phi$ fixed comes from the neighborhood of a point in the $k_i$ plane, say, $k_{i,\alpha}$, where the phase is stationary. For an arbitrary function $f(k_i)$ we then obtain the asymptotic approximation

$$
\int d^2k_i \frac{1}{k_{i,\alpha} \nu_i} f(k_i) \exp[i(k_i \cdot r + k_{i,\alpha} |z|)] \\
= -\frac{2\pi i}{r} f(k_{i,\alpha}) \exp(in_itkr), \quad i = 1 \text{ or } 3, \quad (20)
$$

with $\nu_i$ given by Eq. (7) and $k_{i,\alpha} = k_{i,\alpha} \sin \theta_e \phi$. Vector $e_\phi$ is the radial unit vector in the $xy$ plane corresponding to the direction of $\hat{r}$, e.g., $e_\phi = e_\theta \cos \phi + e_\phi \sin \phi$. The magnitude of $k_{i,\alpha}$ is $k_{i,\alpha} = k_{i,\phi} \sin \theta$ and since in medium $n_i = n_i \sin \theta$, we have $\nu_i = n_i \nu_0$ (i.e., we see that $z$ component of the corresponding traveling plane wave equals $k_{i,\alpha} \sin \theta$.

The wave vector of this partial wave in the angular spectrum is therefore $k_{i,\phi} \hat{r}$. This yields the clear interpretation that the main contribution to the field comes from the traveling plane wave that travels exactly in the direction $\hat{r}$ of the detector.

The far-field approximation to Eq. (18) is found to be

$$
E(r) = \frac{k_0^2}{4\pi \varepsilon_0 r} \exp[i(n_1(k_0r - h \cos \theta)] \\
\times [(d \cdot e_\phi)e_\phi + (d \cdot e_\theta)e_\theta] \\
+ \frac{k_0^2}{4\pi \varepsilon_0 r} \exp[i(n_1(k_0r + h \cos \theta)] \{R_s(\alpha_o) \\
\times (d \cdot e_\phi)e_\phi - R_p(\alpha_o)(d \cdot e_\theta) + 2 \sin \theta (d \cdot e_\phi)e_\theta], \quad (21)
$$

in terms of the spherical unit vectors $e_\phi$ and $e_\theta$. We shall write an equal sign instead of $\approx$. The Fresnel reflection coefficients have to be evaluated at the value of $\alpha$ at the critical point, which is $\alpha_o = k_{i,\alpha}/k_o = n_1 \sin \theta$.

The first term on the right-hand side is the source wave, which travels directly from the dipole toward the detector, and the second term is the reflected wave. The difference in travel distance is seen to be $2H \cos \theta$, indicating that the reflected wave seems to come from a mirror image of the dipole at a distance $H$ below the interface $z = 0$ and

on the $z$ axis. This is even clearer if we write the part with the reflection coefficients as

$$
R_s(\alpha_o)(d \cdot e_\phi)e_\phi - R_p(\alpha_o)(d \cdot e_\theta) + 2 \sin \theta (d \cdot e_\phi)e_\theta \\
= -R_s(\alpha_o)(d \cdot e_\phi)e_\phi + R_p(\alpha_o)(d \cdot e_\theta)e_\theta, \quad (22)
$$

where the mirror dipole $\hat{d}$ is defined as

$$
\hat{d} = \hat{d}_r - \hat{d}_i, \quad (23)
$$

given that $d = d_r + d_i$. In the case of a perfectly conducting substrate, we would have $R_s = -1$ and $R_p = 1$ for every angle of incidence, showing even more the resemblance to the source term.

For the transmitted field we find the asymptotic approximation to be [from Eq. (19)]

$$
E(r) = \frac{k_0^2 n_3 \cos \theta}{4\pi \varepsilon_0 r} \exp[i(k_o n_3 r + h \nu_{1,0})] \frac{1}{\nu_{1,0}} \\
\times \left[T_s(\alpha_o)(d \cdot e_\phi)e_\phi - \frac{1}{n_1} R_p(\alpha_o) \\
\times (\nu_{1,0} d \cdot e_\phi + n_3 \sin \theta (d \cdot e_\phi)e_\theta]. \quad (24)
$$

Here the value of $\alpha$ in the critical point is $\alpha_o = n_3 \sin \theta$, and the Fresnel transmission coefficients have to be evaluated at this $\alpha_o$. Also, the parameter $\nu_{1,0}$ at the critical point appears in this result:

$$
\nu_{1,0} = \sqrt{n_1^2 - n_3^2 \sin \theta}. \quad (25)
$$

It seems that for certain $\theta$, the factor $1/\nu_{1,0}$ in Eq. (24) could present a problem. However, the transmission coefficients $T_{\phi}(\alpha)$ are proportional to $\nu_{1,0}$ [Eqs. (A5) and (A6)], and therefore the $1/\nu_{1,0}$ in Eq. (24) cancels exactly.

5. INTENSITY DISTRIBUTION

The magnetic field for $r$ large can be obtained from Eqs. (21) and (24) by taking the curl, as in Eq. (5), although it appears easier to take the curl in Eqs. (18) and (19), and then make the asymptotic approximations for the resulting angular spectrum representations of the magnetic field. We thus obtain the relation

$$
B(r) = \frac{n_i}{c} \hat{r} \times E(r), \quad i = 1 \text{ or } 3. \quad (26)
$$

The Poynting vector

$$
S(r) = \frac{1}{2 \mu_o} \text{Re} E(r) \times B(r)^*, \quad (27)
$$

and the power per unit solid angle, $dP/d\Omega = r^2 S(r) \cdot \hat{r}$, can then be evaluated for the far field with the results from Section 4.

For the dipole moment we write $d = ud$, with $d$ complex and the unit vector $u$ normalized as $u \cdot u^* = 1$. The intensity distribution will be normalized as

$$
\frac{dP}{d\Omega} = P_0 A(\theta, \phi), \quad (28)
$$

with
the same \( \mathbf{k} \), and thereby the same \( \alpha \), we see that \( \alpha = n_1 \sin \theta_{\text{inc}} = n_3 \sin \theta_2 \), relating the angle of incidence \( \theta_{\text{inc}} \) and the angle of transmission \( \theta_2 \). For detection in \( z < -L \), angle \( \theta_i \) is in the range \( 0 \leq \theta_i < \pi/2 \). Given \( \theta_i \), the angle of incidence follows from \( \sin \theta_{\text{inc}} = (n_3/n_1) \sin \theta_i \). But if \( n_3 > n_1 \), this equation does not necessarily have a solution. Apparently, there exists a transmission angle \( \theta_{\text{ac}}^{(1)} \), given by

\[
\sin \theta_{\text{ac}}^{(1)} = n_1/n_3, \tag{32}
\]

which has the significance that if \( \theta_{\text{ac}}^{(1)} < \theta_i < \pi/2 \), there is no corresponding \( \theta_{\text{inc}} \). However, there is a corresponding \( \alpha \) and therefore a corresponding incident wave. For \( \theta_{\text{ac}}^{(1)} < \theta_i < \pi/2 \), the values of \( \alpha \) are in the range \( n_1 < \alpha < n_3 \), since \( \alpha = n_3 \sin \theta_i \), and this is in the evanescent region of the angular spectrum of the dipole. Consequently, any radiation that is detected at angle \( \theta_i \) in the range \( \theta_{\text{ac}}^{(1)} < \theta_i < \pi/2 \) originates from evanescent waves. From a different point of view, when angle \( \theta_i \) is increased from zero, the angle of incidence also increases, up to the point where it reaches \( \pi/2 \). At this point, \( \theta_i = \theta_{\text{ac}}^{(1)} \), and a further increase of \( \theta_i \) then yields a corresponding evanescent incident wave. This situation is exactly the opposite of total internal reflection, where the angle of incidence \( \theta_{\text{inc}} \) reaches a critical value \( \theta_{\text{c}}^{(1)} \), given by Eq. (2), beyond which the transmitted wave becomes evanescent. Therefore we call \( \theta_{\text{ac}}^{(1)} \) the anti-critical angle.

For detection in \( z < -L \), the dependence on the distance \( h \) between the surface and the dipole is through the factor \( \exp[2h \sin \theta_i \cos \theta_i \sin \theta \cos \theta_i \sin \theta_2 \cos \theta_2] \) in Eq. (31). Since \( \sin \theta = \sin \theta_i \), we see that this factor equals unity for \( 0 \leq \theta_i \leq \theta_{\text{ac}}^{(1)} \), and therefore there is no \( h \) dependence in the radiation pattern. In this range the corresponding dipole waves are traveling. Since traveling waves have the same amplitude everywhere, the travel distance between the dipole and the surface is irrelevant. On the other hand, for \( \theta_i > \theta_{\text{ac}}^{(1)} \) this factor gives an exponential dependence on \( h \), reflecting the fact that the corresponding evanescent dipole waves decay exponentially from some finite value at \( z = H \) to zero in the negative \( z \) direction. As a result, the detected power for \( \theta_i > \theta_{\text{ac}}^{(1)} \) diminishes rapidly with increasing \( h \). This is illustrated in Fig. 4, where \( h = 4 \pi \), and it is seen that almost no radiation appears in \( \theta_i > \theta_{\text{ac}}^{(1)} \).
7. POWER OF THE EVANESCENT WAVES

For the radiation in \( z > H \) we have \( 0 < \theta < \pi/2 \), and the corresponding range of \( \alpha \) is \( 0 < \alpha < n_1 \), because \( \alpha = n_1 \sin \theta_\text{inc} \) and \( \theta = \theta_\text{inc} \). This shows that exactly all traveling dipole waves contribute to the radiation in \( z > H \), and there is no contribution from evanescent waves here. For detection in \( z < -L \) the range of \( \alpha \) is \( 0 < \alpha < n_3 \), since \( \alpha = n_3 \sin \theta_\text{inc} \). If \( n_3 < n_1 \), the corresponding dipole waves are traveling, and there is no contribution from evanescent waves. For values of \( \alpha \) in the range \( n_3 < \alpha < n_1 \), there are still traveling dipole waves incident on the surface \( z = 0 \), but the angle of incidence is larger than the critical angle \( \theta_\text{c}^{(1)} \), Eq. (2), and therefore the transmitted waves in medium \( n_3 \) are evanescent, and they do not appear in the far field as radiation. Only for \( n_3 > n_1 \) can we have evanescent dipole waves appearing in the far field, in \( z < -L \), which are converted by the layer into traveling waves.

In order to quantify the relative contribution of the evanescent waves to the power transmitted through the layer, we consider the integrated power. To this end, we first notice that the \( \phi \) dependence of the intensity distribution \( A(\theta, \phi) \) is purely geometrical since it enters only through the unit vectors \( e_\theta \), \( e_\phi \), and \( e_z \). The \( \phi \) dependence, on the other hand, is essential, as follows from the fact that \( \sin \theta \) appears in the arguments of the Fresnel coefficients. Another \( \theta \) dependence enters through \( d\Omega = \sin \theta d\theta d\phi \). We therefore introduce

\[
B(\theta) = \sin \theta \int_0^{2\pi} d\phi A(\theta, \phi),
\]

in terms of which the power per unit polar angle, emitted in the \( \theta \) direction, becomes

\[
\frac{dP}{d\theta} = P_o B(\theta).
\]

Performing the integrations over \( \phi \) then yields

\[
B(\theta) = \frac{3}{2} n_1 \sin \theta (1 - |u_z|^2) \\
\times \left[ [1 + R_\phi(n_1 \sin \theta) \exp(2in_1 \cos \theta)]^2 + \cos^2 \theta [1 - R_\phi(n_1 \sin \theta) \exp(2in_1 \cos \theta)]^2 \right] \\
+ \frac{3}{2} n_1 \sin^3 \theta |u_z|^2 \\
\times \left[ 1 + R_\phi(n_1 \sin \theta) \exp(2in_1 \cos \theta) \right]^2,
\]

for \( z > H \), and

\[
B(\theta) = \frac{3n_3^3 \sin \theta \cos^2 \theta}{8n_1^2 |n_1^2 - n_3^2 \sin^2 \theta|} \left[ (1 - |u_z|^2) \\
\times [(n_3^2)^2 T_\phi(n_3 \sin \theta)]^2 + |n_1^2 - n_3^2 \sin^2 \theta| [T_\phi(n_3 \sin \theta)]^2 \right] \\
+ |u_z|^2 n_3^2 \sin^2 \theta \left[ [T_\phi(n_3 \sin \theta)]^2 \right] \\
\times \exp(-2\delta \Im(n_1^2 - n_3^2 \sin^2 \theta) |u_z|^2),
\]

for \( z < -L \). It is interesting to notice that the dependence on the dipole orientation vector \( \mathbf{u} \) enters only as \( |u_z|^2 \), with \( 0 < |u_z|^2 < 1 \), in contrast to the result for \( A(\theta, \phi) \), which depends on the three Cartesian components of \( \mathbf{u} \) separately.

The transmitted power due to traveling dipole waves ends up in the cone \( 0 < \theta < \theta_\text{c}^{(1)} \) and is given by

\[
P_{\text{tr}} = P_o \int_0^{\pi/2} d\theta B(\theta),
\]

whereas the contribution from evanescent waves is

\[
P_{\text{ev}} = P_o \int_{\pi/2}^{\pi - \theta_\text{c}^{(1)}} d\theta B(\theta).
\]

As a measure for the relative contribution of the evanescent waves, we define

\[
f = \frac{P_{\text{ev}}}{P_{\text{ev}} + P_{\text{tr}}} \times 100\%.
\]

The quantities \( P_{\text{ev}} \) and \( P_{\text{tr}} \) are computed by numerical integration. For parameters as in Fig. 4, with \( h = 4\pi \), the value of \( f \) is found to be 0.56%, indicating that only a very small fraction of the power is due to evanescent waves in this case. An example of just the opposite situation is illustrated in Fig. 5, where \( h = 0 \). Here we find \( f = 95\% \), and the figure clearly shows that nearly all intensity is emitted at a transmission angle larger than \( \theta_\text{c}^{(1)} \).

In order to illustrate the significance of the evanescent waves, we consider a single \( n_1 - n_3 \) interface (this affects only the Fresnel coefficients, which simplify considerably) and \( h = 0 \). Then it can be shown that \( P_{\text{ev}} \) and \( P_{\text{tr}} \) are functions of \( n_1/n_3 \) only, with both an overall factor of \( n_3 \).

For a dipole embedded in a medium with index of refraction \( n_3 \), the total power emitted in all directions is \( n_3 P_o \). Figure 6 shows \( P_{\text{ev}}/(n_3 P_o), P_{\text{tr}}/(n_3 P_o) \), and \( f \) as functions of \( n_1/n_3 \) for a dipole in the xy plane. Notice that the independent variable \( n_1/n_3 \) equals \( \sin \theta_\text{c}^{(1)} \). At approximately \( n_1/n_3 \approx 0.8 \), we have \( P_{\text{ev}} \approx P_{\text{tr}} \), and for \( n_3 > 1.25 n_1 \) the power of the evanescent waves exceeds the power of the traveling waves. Figure 7 shows the...
same graphs for a dipole along the z axis. Here we see that \(P_{ev}/(n_3P_o)\) falls to zero for \(n_1/n_3 \rightarrow 0\).

For \(n_3 \gg n_1\) we have \(r \rightarrow 0\), and with \(\eta(0) = 0\) this gives \(P_{ev} = n_3P_o\), showing that for a dense medium \(n_3\), relative to \(n_1\), the power of the evanescent waves is the same as the power of a free dipole in medium \(n_3\), for which all waves are traveling. It should be noted that this conclusion strictly holds for \(h = 0\) only. The \(h\) dependence of \(B(\theta)\) in Eq. (36) comes in through the factor \(\exp[-2h \text{Im}n_1^2/(n_3^2 - n_3^2 \sin^2 \theta)^{1/2}]\), which can also be written as \(\exp[-2h n_3 \text{Im} \sin^2 \theta_{ac}^{(1)} - \sin^2 \theta_{ac}^{(2)}^{1/2}]\). Since \(\sin \theta_{ac}^{(1)} < \sin \tilde{\theta}_{ac} < \sin \theta_{ac}^{(2)}\), the exponent is nonzero; and for any finite \(h\), an increasing \(n_3\) makes this factor vanish. Therefore \(P_{ev}/(n_3P_o) \rightarrow 0\) for \(h \neq 0\) and \(r \rightarrow 0\). This feature is illustrated in Fig. 8.

8. ANTI-CRITICAL ANGLE OF THE SECOND KIND

The anti-critical angle of the first kind is the transmission angle at which the incident wave turns evanescent. We now consider the waves in the layer with index of refraction \(n_2\). When these are traveling waves with angle \(\theta_2\) with respect to the normal, then we have \(\sin \theta_2 = (n_2/n_3)\sin \theta_1\), given \(\theta_1\). When \(n_3 > n_2\), \(\theta_2\) reaches \(\pi/2\) at the transmission angle \(\theta_{ac}^{(2)}\) given by

\[
\sin \theta_{ac}^{(2)} = n_2/n_3. \tag{42}
\]

Then for \(\theta_{ac}^{(2)} < \theta_t < \pi/2\) the waves in the layer are evanescent, which greatly reduces the transmission with increasing \(L\). It should be noted that this phenomenon is independent of the nature of the incident dipole waves. Also, we can have two anti-critical angles, with either \(\theta_{ac}^{(1)} < \theta_{ac}^{(2)}\) or \(\theta_{ac}^{(1)} > \theta_{ac}^{(2)}\), or we can have only \(\theta_{ac}^{(1)}\) or only \(\theta_{ac}^{(2)}\), or none at all, depending on the relative values of \(n_1\), \(n_2\), and \(n_3\).

Figure 9 shows the effect of the second anti-critical angle for the case of \(\theta_{ac}^{(1)} < \theta_{ac}^{(2)}\). For \(0 < \theta_t < \theta_{ac}^{(1)}\), the dipole, layer and transmitted waves are all traveling. For \(\theta_{ac}^{(1)} < \theta_t < \theta_{ac}^{(2)}\), the dipole waves are evanescent but the waves in the layer are still traveling, and for \(\theta_{ac}^{(2)} < \theta_t < \pi/2\) both the dipole and the layer waves are eva-
nescent, with, of course, the transmitted waves still traveling. In Fig. 9 we have \( l = 2\pi, \) with \( l = k_L L \) the dimensionless layer thickness, so we have a layer thickness of one wavelength of the incident radiation \( (n_1 = 1) \). We see that the transmission in \( \theta_{\mathrm{ac}}^{(2)} < \theta_l < \pi/2 \) has already become negligible for this relatively small value of \( l \). It is interesting to notice that in the region \( \theta_{\mathrm{ac}}^{(1)} < \theta_l < \theta_{\mathrm{ac}}^{(2)} \) there seems to be an interference structure. This cannot be the case, however, because for a given angle \( \theta_l \) there is only a single wave, according to the stationary-phase interpretation. From Eqs. (A5) and (A6) we see that the Fresnel transmission coefficients have a factor of \( \exp(2i n_r L \ell) \) in the denominator. This can also be written as \( \exp[2in_r L (\sin^2 \theta_{\mathrm{ac}}^{(1)} - \sin^2 \theta_{\mathrm{ac}}^{(2)})] \). For \( \theta_{\mathrm{ac}}^{(2)} < \theta_l \), the exponent is imaginary, leading to oscillations with varying \( \theta_l \). This, of course, is a reflection of the interference of the waves with wave vectors \( \mathbf{k}_r \) and \( \mathbf{k}_l \), representing the traveling waves in the layer [Fig. 2(a)], and this affects the Fresnel transmission coefficients.

9. CONCLUSIONS

Dipole radiation is a superposition of traveling and evanescent plane waves when represented by an angular spectrum. For a dipole embedded in a medium with index of refraction \( n_1 \), only the traveling waves contribute to the radiation in the far field, and the evanescent waves remain unobserved. When the radiation passes through an interface or a layer, as in Fig. 1, a portion of the evanescent waves will be converted into traveling waves if \( n_3 > n_1 \), and they become observable in the far field. We have shown that there exists a transmission angle \( \theta_{\mathrm{ac}}^{(1)} \) that has the significance that any radiation detected in \( \theta_{\mathrm{ac}}^{(1)} < \theta_l < \pi/2 \) has its origin in evanescent dipole waves. Partial angular spectrum waves with their \( \mathbf{k}_r \) vector in the ring \( k_r n_1 < k_l < k_r n_3 \) in the \( \mathbf{k}_r \) plane contribute to this phenomenon, and the observation direction \( \mathbf{r} \) of the emanating traveling wave determines uniquely the corresponding \( \mathbf{k}_l \) in this ring. In this fashion, the evanescent field of the dipole can be partially observed in the far field, and this has been seen experimentally.\(^{18} \) In more-contemporary measurements, the sample is placed in air or in an aqueous solution with low index of refraction on an oil-immersion hemispherical lens. In this geometry, the radiation in the far field exits the lens under normal incidence, and the intensity distribution should greatly resemble the results presented in this paper for \( z < 0 \).

We have also shown that the total power in the far field due to evanescent dipole waves can greatly exceed the contribution of the traveling waves \( (f = 95\% \) in Fig. 5) and that this power can exceed the power by a free dipole in medium \( n_3 \). Finally, when transmission through a layer is considered, a second anti-critical transmission angle \( \theta_{\mathrm{ac}}^{(2)} \) appears when \( n_3 > n_2 \). For \( \theta_l > \theta_{\mathrm{ac}}^{(2)} \), the waves in the layer become evanescent, and hardly any radiation will appear in this region of observation when the layer thickness \( L \) is not too small.

APPENDIX A

The Fresnel reflection and transmission coefficients depend on the three indices of refraction, the dimensionless layer thickness \( \ell = k_r L \), and the variable \( \alpha \), representing in dimensionless form the value of \( k_1 \). These Fresnel coefficients are most conveniently expressed in terms of the three \( v_i \) from Eq. (7). With the notation

\[
\Lambda_\alpha = (v_1 + v_2)(v_2 + v_3) + (v_1 - v_2)(v_2 - v_3) \times \exp(2i v_2 \ell), \quad (A1)
\]

\[
\Lambda_p = (n_2^2 v_1 + n_1^2 v_2)(n_3^2 v_2 + n_2^2 v_3) + (n_2^2 v_1 - n_1^2 v_2)(n_3^2 v_2)
- n_2^2 v_3 \exp(2i v_2 \ell), \quad (A2)
\]

we have

\[
R_\alpha(\alpha) = \frac{1}{\Lambda_\alpha} \left[ (v_1 - v_2)(v_2 + v_3) + (v_1 + v_2)(v_2 - v_3) \times \exp(2i v_2 \ell) \right], \quad (A3)
\]

\[
R_p(\alpha) = \frac{1}{\Lambda_p} \left[ (n_2^2 v_1 - n_1^2 v_2)(n_3^2 v_2 + n_2^2 v_3) + (n_2^2 v_1
+ n_1^2 v_2)(n_3^2 v_2 - n_2^2 v_3) \exp(2i v_2 \ell) \right], \quad (A4)
\]

\[
T_\alpha(\alpha) = \frac{4v_1 v_2}{\Lambda_\alpha} \exp(i(v_2 - v_3)\ell), \quad (A5)
\]

\[
T_p(\alpha) = \frac{4v_1 v_2 n_1^2 n_2^2 n_3^2}{\Lambda_p} \exp[i(v_2 - v_3)\ell]. \quad (A6)
\]

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