Spatial separation of the traveling and evanescent parts of dipole radiation

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Electric dipole radiation consists of traveling and evanescent plane waves. When radiation is detected in the far field, only the traveling waves will contribute to the intensity distribution, as the evanescent waves decay exponentially. We propose a method to spatially separate the traveling and evanescent waves before detection. It is shown that when the radiation passes through an interface, evanescent waves can be converted into traveling waves and can subsequently be observed in the far field. Let the radiation be observed under angle θ_t with the normal. Then there exists an angle θ_{ac} such that for $0 \le \theta_t < \theta_{\mathrm{ac}}$ all intensity originates in traveling waves, whereas for $\theta_{\mathrm{ac}} < \theta_t < \pi/2$ only evanescent waves contribute. It is shown that with this technique and under the appropriate conditions almost all far-field power can be provided by evanescent waves. © 2003 Optical Society of America

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When an electric dipole embedded in a medium with index of refraction n_1 oscillates with angular frequency ω, it emits spherical waves with wave number $k = k_0 n_1$, where $k_0 = \omega/c$. This dipole radiation can also be considered a superposition of plane waves, each of which satisfies Maxwell's equations individually. Such a representation requires that we adopt a preferred direction in space, which we shall take as the z axis. Then the radiation field consists of plane waves that travel in all directions, and, in addition, evanescent plane waves appear. These evanescent waves travel along the xy plane and decay exponentially in the positive and negative z directions. Because of this decay the evanescent waves contribute predominantly to the near field and vanish at a normal distance of approximately a wavelength from the xy plane. When this dipole radiation is detected with a macroscopic device in the far field, only the traveling waves will contribute to the observed intensity. In this Letter we propose an experimentally feasible method for detecting the existence of evanescent waves in the dipole field by measurement in the far field.

We consider the situation in which a dipole, with dipole moment $\mathbf{d}(t) = \mathrm{Re}[\mathbf{d}\exp(-i\omega t)]$, is located on the z axis at a distance H from the xy plane and is surrounded by a medium with index of refraction n_1. The superposition of plane waves for the electric part of the dipole (source) field is given by $\mathbf{E}_S(\mathbf{r},t) = \mathrm{Re}[\mathbf{E}_S(\mathbf{r})\exp(-i\omega t)]$, where[1,2]

$$\mathbf{E}_S(\mathbf{r}) = \frac{i}{8\pi^2\epsilon_0 {n_1}^2}\int \mathrm{d}^2\mathbf{k}_\parallel \frac{1}{\beta}[{k_0}^2{n_1}^2\mathbf{d} - (\mathbf{d}\cdot\mathbf{k})\mathbf{k}] \times \exp[i\mathbf{k}\cdot(\mathbf{r} - H\mathbf{e}_z)]. \qquad (1)$$

This representation is usually referred to as the angular spectrum representation. Here $\mathbf{k}_\parallel$ is a vector parallel to the xy plane, and the integration runs over the entire $\mathbf{k}_\parallel$ plane. For a given $\mathbf{k}_\parallel$, wave vector $\mathbf{k}$ of a partial wave is $\mathbf{k} = \mathbf{k}_\parallel + \beta\,\mathrm{sgn}(z - H)\mathbf{e}_z$ in terms of the parameter

$$\beta = \begin{cases} ({k_0}^2{n_1}^2 - {k_\parallel}^2)^{1/2} & k_\parallel < k_0 n_1 \\ i({k_\parallel}^2 - {k_0}^2{n_1}^2)^{1/2} & k_\parallel > k_0 n_1 \end{cases}. \qquad (2)$$

This shows that for $0 \le k_\parallel < k_0 n_1$ the plane wave is traveling and for $k_\parallel > k_0 n_1$ the wave is evanescent.

We now consider the configuration shown in Fig. 1, where the region $z < 0$ is occupied by a medium with index of refraction n_2, and we assume that $n_2 > n_1$. Source field $\mathbf{E}_S(\mathbf{r})$ contains waves that travel and decay in the positive z direction and waves that travel and decay toward the xy plane. The latter will be partially reflected at the surface and partially transmitted, the details of which are accounted for by the appropriate Fresnel coefficients. The total field in $z > H$, $0 < z < H$, and $z < 0$ then again assumes the form of an angular spectrum, where each specular wave in $z > 0$ has a Fresnel reflection coefficient as an additional amplitude factor and the amplitude of

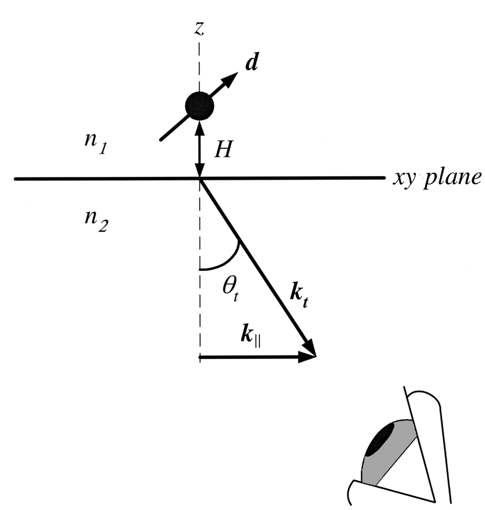

Fig. 1. Illustration of the setup and wave vectors $\mathbf{k}_t$ and $\mathbf{k}_\parallel$ for a given observation angle θ_t.

each transmitted wave contains a Fresnel transmission coefficient.[1,3]

With the method of stationary phase,[4] expressions for the far field can be obtained. For the region $z > 0$ it yields the asymptotic form of the source field [Eq. (1)] in combination with specularly reflected waves, leading to interference and a lobe structure in the intensity distribution.[5,6] Only traveling waves contribute to the far field in this region, because the evanescent waves decay rapidly in the direction away from the surface.

The far field in the region $z < 0$ involves only transmitted waves, and we note that effectively only one partial wave from the angular spectrum contributes for a given observation direction, as is implicit in the method of stationary phase (the traveling wave corresponding to the critical point in the $\mathbf{k}_\parallel$ plane). Therefore there is no interference between waves, as there is in the region $z > 0$. As indicated in Fig. 1, angle of transmission θ_t is the observation direction for a given traveling wave, and this is also the direction of the corresponding wave vector, $\mathbf{k}_t$. Because n_2 is the index of refraction in this medium, the wave number is $k_t = k_0 n_2$, and it follows from Fig. 1 that the corresponding $\mathbf{k}_\parallel$ vector has a magnitude $k_\parallel = k_0 n_2 \sin\theta_t$. To satisfy the boundary conditions at the xy plane, both the source wave and the corresponding transmitted wave (and the reflected wave) must have the same $\mathbf{k}_\parallel$. This means that observation angle θ_t determines the value of $k_\parallel$, and, with Eq. (2), this in turn gives the value of β:

$$\beta = \begin{cases} k_0(n_1{}^2 - n_2{}^2 \sin^2\theta_t)^{1/2} & n_2 \sin\theta_t < n_1 \\ ik_0(n_2{}^2 \sin^2\theta_t - n_1{}^2)^{1/2} & n_2 \sin\theta_t > n_1 \end{cases}. \quad (3)$$

We then see that, depending on θ_t, the source wave can be either traveling or evanescent, even though the transmitted wave is traveling for all directions θ_t. A similar situation arises for the problem of Čerenkov radiation emitted near an interface. Evanescent components of this radiation can emerge as traveling, after transmission through the interface.[7,8]

Evidently, there exists an angle θ_{ac} for which $n_2 \sin\theta_{ac} = n_1$, e.g.,

$$\sin\theta_{ac} = n_1/n_2\,. \quad (4)$$

Then the source wave (and the reflected wave) is traveling for $0 \le \theta_t < \theta_{ac}$ and evanescent for $\theta_{ac} < \theta_t < \pi/2$. At this angle θ_{ac} we have $k_\parallel = k_0 n_2 \sin\theta_{ac}$, and this is $k_\parallel = k_0 n_1$, which then gives $k_\parallel = k$. Therefore, at observation angle θ_{ac}, the source wave has an angle of incidence of $\theta_i = \pi/2$ and is on the borderline between traveling and evanescent. This is just the opposite situation as for the regular critical angle, $\sin\theta_c = n_2/n_1$, where for $\theta_i > \theta_c$ the transmitted wave becomes evanescent whereas the incident wave remains traveling. We therefore call θ_{ac} the anticritical angle. The maximum value of θ_t is $\pi/2$, which gives $k_\parallel = k_0 n_2$. This shows that evanescent source waves in the ring $k_0 n_1 < k_\parallel < k_0 n_2$ of the $\mathbf{k}_\parallel$ plane contribute to radiation in $\theta_{ac} < \theta_t < \pi/2$, whereas evanescent waves of the dipole with larger $k_\parallel$ never contribute to the far field. To conclude this observation, there exists an angle θ_{ac} such that for any radiation detected in the cone $0 \le \theta_t < \theta_{ac}$ the original source waves are traveling and any radiation (in $z < 0$) outside this cone was emitted by the dipole as evanescent waves. In this fashion, the interface spatially separates the traveling and evanescent parts of the dipole radiation, and any observation of radiation in the region $\theta_{ac} < \theta_t < \pi/2$ would confirm the existence of evanescent waves in dipole radiation. It is interesting to note that this phenomenon is independent of the normal distance H between the dipole and the surface and is independent of the orientation of dipole vector $\mathbf{d}$.

From the expression for the electric field in the far zone we can compute the Poynting vector and the intensity distribution (power per unit solid angle, $dP/d\Omega$). Figure 2 shows a polar diagram of a typical intensity distribution $dP/d\Omega$, in units of $P_0 = (\mathbf{d} \cdot \mathbf{d}^*\omega^4)/(32\pi^2\epsilon_0 c^3)$. Here we have taken $n_1 = 1$ and $n_2 = \sqrt{2}$, which give $\theta_{ac} = 45°$, and a dipole moment that rotates in the xy plane (such that the intensity distribution is cylindrically symmetric about the z axis). The anticritical angle is indicated by the dashed line in the figure. The distance between the dipole and the surface was taken to be two wavelengths, and we see that outside the cone $0 \le \theta_t < \theta_{ac}$ there is no noticeable contribution to the intensity. As a measure of the contribution of the evanescent waves we introduce the quantity

$$f = \frac{P_{ev}}{P_{tr} + P_{ev}} \times 100\%\,, \quad (5)$$

where P_{ev} and P_{tr} are the power per unit solid angle, integrated over the solid angles that correspond to $\theta_{ac} < \theta_t < \pi/2$ and $0 \le \theta_t < \theta_{ac}$, respectively. With numerical integration we found $f = 1.43\%$ for the parameters of Fig. 2. The reason that this value

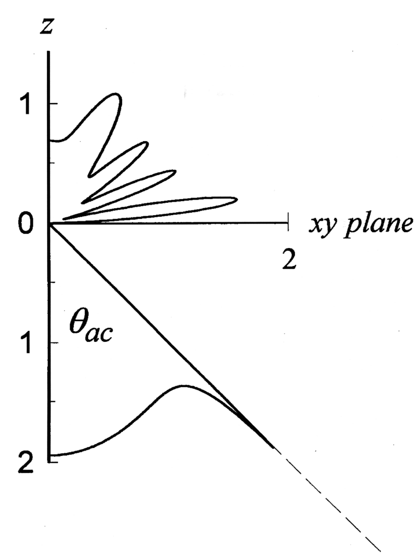

Fig. 2. Polar diagram of the intensity distribution $dP/d\Omega$ of the radiation, in units of P_0, emitted by a dipole located two wavelengths above the surface and rotating in the xy plane. The indices of refraction are $n_1 = 1$ and $n_2 = \sqrt{2}$. The dashed line indicates the anticritical angle θ_{ac}.

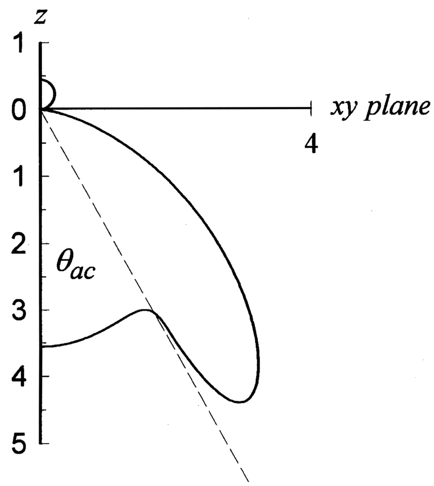

Fig. 3. Intensity distribution for a dipole rotating in the xy plane and very close to the surface ($H = 0$). The anticritical angle is 30°, and the indices of refraction are $n_1 = 1$ and $n_2 = 2$.

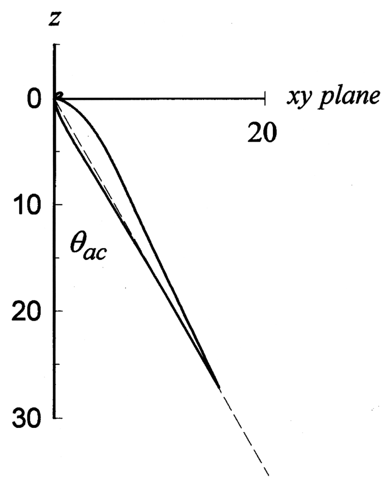

Fig. 4. Intensity distribution for the same parameters as in Fig. 3 but for a linear dipole in the z direction.

is so small is that the evanescent waves decay exponentially over a typical distance of approximately a wavelength and therefore they have already died out when they reach the surface. For Fig. 3 we took $H = 0$ and $n_2 = 2$, with everything else the same. Here $\theta_{\mathrm{ac}} = 30°$, and now we obtain $f = 80.3\%$, indicating that most of the radiation in the far field has its origin in evanescent dipole waves. Figure 4 illustrates the radiation pattern for a dipole that is oriented perpendicular to the surface, with the other parameters the same as in Fig. 3, and we observe that most of the radiation intensity emerges from the interface in the direction $\theta_t \approx \theta_{\mathrm{ac}}$. The fraction f for this case is 86.4%.

As for possible experimental verification of the predicted results, first we note that the medium with index of refraction n_2 should have a thickness of many wavelengths at least, because otherwise the far-field approximation is not valid. The detector should scan the far-field intensity over half a sphere below the xy plane. When the dipole is several wavelengths away from the interface, no radiation should be observed in the region $\theta_{\mathrm{ac}} < \theta_t < \pi/2$, because the evanescent waves from the dipole die out before reaching the interface. When the distance H between the dipole and the surface is decreased to a fraction of a wavelength, with all other parameters the same, radiation intensity should appear in the region $\theta_{\mathrm{ac}} < \theta_t < \pi/2$, giving evidence of the fact that this radiation originates in evanescent dipole waves.

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