Traveling and evanescent parts of the electromagnetic Green's tensor

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The angular spectrum representation of the electromagnetic Green's tensor has a part that is a superposition of exponentially decaying waves in the $+z$ and $-z$ directions (evanescent part) and a part that is a superposition of traveling waves, both of which are defined by integral representations. We have derived an asymptotic expansion for the z dependence of the evanescent part of the Green's tensor and obtained a closed-form solution in terms of the Lommel functions, which holds in all space. We have shown that the traveling part can be extracted from the Green's tensor by means of a filter operation on the tensor, without regard to the angular spectrum integral representation of this part. We also show that the so-called self-field part of the tensor is properly included in the integral representation, and we were able to identify this part explicitly.

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1. INTRODUCTION

Emission of electromagnetic radiation by a localized current density $\mathbf{j}(\mathbf{r}, t)$ is most conveniently described by means of a Green's tensor $\vec{\vec{g}}(\mathbf{r}, \omega)$. The emitted electric field $\mathbf{E}(\mathbf{r}, t)$ is assumed to have a temporal Fourier transform $\hat{\mathbf{E}}(\mathbf{r}, \omega)$; e.g., the field is represented as

$$\mathbf{E}(\mathbf{r}, t) = \frac{1}{\pi}\mathrm{Re}\int_0^\infty \mathrm{d}\omega \hat{\mathbf{E}}(\mathbf{r}, \omega)\exp(-i\omega t), \qquad (1)$$

and a similar expression holds for the current density. The solution of Maxwell's equations can then be written in the form (details in Section 2)

$$\hat{\mathbf{E}}(\mathbf{r}, \omega) = \frac{i\omega\mu_o}{4\pi}\int \mathrm{d}^3\mathbf{r}'\vec{\vec{g}}(\mathbf{r} - \mathbf{r}', \omega) \cdot \hat{\mathbf{j}}(\mathbf{r}', \omega). \qquad (2)$$

For atomic or molecular sources, the main interest is usually in the properties of the solution in the radiation (far) zone, since that is where the field is measured with detectors of macroscopic size. A common approach is to expand the Green's tensor in multipole fields, leading to the characteristic angular distribution patterns for the radiated power. However, in the presence of boundaries this method becomes cumbersome as a consequence of the fact that the multipole fields have a very specific spatial dependence, which will not be compatible with the boundary conditions at hand. With the rapidly developing experimental techniques in near-field (nano-scale) optics,[1–6] it has become feasible to detect radiation with a spatial resolution of approximately an optical wavelength in the vicinity of a source, which is typically located near a dielectric or metallic medium. This situation necessitates that different representations of the solution of Maxwell's equations that are better adapted to the study of electromagnetic radiation in the neighborhood of the radiating source have to be employed.

When the source is located near a dielectric medium, the emitted radiation will partially reflect at the interface, and the total field in the neighborhood of the source is the sum of $\hat{\mathbf{E}}(\mathbf{r}, \omega)$ from Eq. (2) and the reflected field, leading to interference. It then seems tempting to adopt a spatial Fourier transform of the Green's function,[7] yielding a superposition of traveling plane waves of the form $\exp[\mathrm{i}(\mathbf{k} \cdot \mathbf{r} - \omega t)]$. If we denote by $k_o = \omega/c$ the wave number of the radiation corresponding to frequency ω, then we note that, in general, waves with $k \neq k_o$ will contribute to the superposition. This implies that the partial waves do not satisfy the free-space Maxwell equations individually, rendering the applicability of this approach limited. A solution to this problem is to adopt a two-dimensional spatial Fourier transform, say, in x and y but not in z, leading to waves of the form $\exp(\mathrm{i}\mathbf{k}_\parallel \cdot \mathbf{r})$, with $\mathbf{k}_\parallel$ a vector in the xy plane. The z component of the wave vector is then chosen is such a way that the dispersion relation $k_o = \omega/c$ is satisfied by each wave individually. To be specific, for a given $\mathbf{k}_\parallel$ and a given k_o, we impose the constraint $k_\parallel^2 + k_z^2 = k_o^2$ on the z component of the wave vector. In this fashion we obtain an expansion in traveling waves with $k_z = \pm(k_o^2 - k_\parallel^2)^{1/2}$, but, in addition, waves with $k_z = \pm\mathrm{i}(k_\parallel^2 - k_o^2)^{1/2}$ appear, in the case that $k_\parallel > k_o$. These are evanescent waves that decay in the z direction and travel in the xy plane. In this construction the partial waves satisfy the free-space Maxwell equations, and each partial wave of the expansion can be considered separately. Especially in the situation of a source near a medium, the surface of which is then taken as the xy plane, this decomposition is extremely useful. The reflection amplitudes are the well-known Fresnel coefficients, and the reflected field becomes a superposition of traveling and evanescent waves. This method has been applied successfully to obtain the radiation field for an electric dipole near a mirror or dielectric[8–11] and near a nonlinear medium.[12]

These superpositions of traveling and evanescent waves are commonly referred to as angular spectrum representations, and they have been studied extensively.[13,14] Previously, the primary goal has been to find the asymptotic far-field solution, which can be obtained with the method of stationary phase.[15,16] More recently,[17,18] attention has shifted to the separate contributions of the traveling and evanescent waves to the angular spectrum. In the far field, obviously the traveling waves dominate, whereas the near field is determined mainly by the evanescent components of the angular spectrum. It is well known, for instance, that the decay rate of an electronic transition (inverse lifetime of an excited state) in an atom located near a medium can acquire a large contribution from evanescent waves owing to the coupling to plasmon modes in the substrate.[19] Such plasmon modes have been studied extensively,[20] and it has been confirmed experimentally[21–22] that indeed a large portion of the atomic decay rate is due to the presence of evanescent waves in the dipole radiation. As for the far field, there has been some controversy in the literature as to whether the evanescent waves contribute at all.[23–26]

In this paper we investigate in more detail the traveling and evanescent parts of the electromagnetic Green's function. After introducing the Green's tensor formalism in Section 2 and its angular spectrum representation in Section 3, we first show how the self-field is represented by the angular spectrum, an issue that has largely been neglected in the literature. We then derive an asymptotic expansion of the evanescent part to exhibit clearly the contribution to the far field. Also, the integrals representing the evanescent field are evaluated analytically for all points in space, which could prove useful for the study of the near field. Finally, it is shown that the traveling part of the tensor can be obtained by means of a filter operation on the tensor.

2. GREEN'S TENSOR

For a given localized current density $\hat{\mathbf{j}}(\mathbf{r}, \omega)$, the solution of Maxwell's equations for the electric field is given by

$$\hat{\mathbf{E}}(\mathbf{r}, \omega) = \frac{i\omega\mu_o}{4\pi}\int d^3\mathbf{r}' g(\mathbf{r}-\mathbf{r}', \omega)\hat{\mathbf{j}}(\mathbf{r}', \omega) + \frac{i\omega\mu_o}{4\pi k_o^2}\nabla\left[\nabla\cdot\int d^3\mathbf{r}' g(\mathbf{r}-\mathbf{r}', \omega)\hat{\mathbf{j}}(\mathbf{r}', \omega)\right], \tag{3}$$

where μ_o is the permeability of free space and $k_o = \omega/c$. Here $g(\mathbf{r}, \omega)$ is the scalar free-space Green's function for the Helmholz equation:

$$g(\mathbf{r}, \omega) = \exp(ik_o r)/r. \tag{4}$$

To cast the solution for $\hat{\mathbf{E}}(\mathbf{r}, \omega)$ into the form of Eq. (2), we need to move the differential operators in the last term on the right-hand side of Eq. (3) under the integral sign. The singularity at $\mathbf{r}' = \mathbf{r}$ then yields an extra term,[27] and when we bring the result into dyadic form, we obtain the result

$$\hat{\mathbf{E}}(\mathbf{r}, \omega) = -\frac{i}{3\epsilon_o\omega}\hat{\mathbf{j}}(\mathbf{r}, \omega) + \frac{i\omega\mu_o}{4\pi}\int d^3\mathbf{r}'\overleftrightarrow{d}(\mathbf{r}-\mathbf{r}', \omega)\cdot\hat{\mathbf{j}}(\mathbf{r}', \omega). \tag{5}$$

The first term on the right-hand side is due to the singularity of the scalar Green's function, and this contribution to the electric field is called the self-field. Usually, this term is omitted, since it is present only inside the source. However, in the near-field optics literature this term has attracted some attention lately,[28,29] and it should be retained for mathematical consistency.[30] The remaining parts are combined in the second term, where the tensor $\overleftrightarrow{d}(\mathbf{r}, \omega)$ is defined by

$$\overleftrightarrow{d}(\mathbf{r}, \omega) = \left(\overleftrightarrow{I} + \frac{1}{k_o^2}\nabla\nabla\right)g(\mathbf{r}, \omega), \tag{6}$$

in terms of the unit tensor $\overleftrightarrow{I}$ and the scalar Green's function. Comparison with Eq. (2) then shows that the electromagnetic Green's tensor $\overleftrightarrow{g}(\mathbf{r}, \omega)$ can be written as

$$\overleftrightarrow{g}(\mathbf{r}, \omega) = -\frac{4\pi}{3k_o^2}\delta(\mathbf{r})\overleftrightarrow{I} + \overleftrightarrow{d}(\mathbf{r}, \omega). \tag{7}$$

3. ANGULAR SPECTRUM

The scalar Green's function $g(\mathbf{r}, \omega)$ can be represented as a two-dimensional Fourier integral,[31]

$$g(\mathbf{r}, \omega) = \frac{i}{2\pi}\int d^2\mathbf{k}_\parallel \frac{1}{\beta}\exp(i\mathbf{k}_\parallel\cdot\mathbf{r} + i\beta|z|), \tag{8}$$

which is commonly referred to as Weyl's representation. The parameter β is defined by

$$\beta = \begin{cases} \sqrt{k_o^2 - k_\parallel^2}, & k_\parallel < k_o \\ i\sqrt{k_\parallel^2 - k_o^2}, & k_\parallel > k_o \end{cases}, \tag{9}$$

which shows that $g(\mathbf{r}, \omega)$ is now written as an integral over traveling and evanescent scalar waves. The Green's tensor $\overleftrightarrow{g}(\mathbf{r}, \omega)$ can be represented in a similar way. The easiest way to derive its representation is probably by use of Eq. (8) and substitution of this result in Eq. (6) for $\overleftrightarrow{d}(\mathbf{r}, \omega)$. Care should be exercised here concerning the self-field contribution. This part in Eq. (7) for $\overleftrightarrow{g}(\mathbf{r}, \omega)$ comes from the singularity in the integrand of the second term on the right-hand side of Eq. (3). However, when we use representation (8) for $g(\mathbf{r}, \omega)$ in Eq. (3), then the singularity at $\mathbf{r}' = \mathbf{r}$ disappears, and the differential operators can be brought freely under the integral. Therefore the proper representation is

$$\overleftrightarrow{g}(\mathbf{r}, \omega) = \frac{i}{2\pi}\int d^2\mathbf{k}_\parallel \frac{1}{\beta}\left(\overleftrightarrow{I} + \frac{1}{k_o^2}\nabla\nabla\right) \times \exp(i\mathbf{k}_\parallel\cdot\mathbf{r} + i\beta|z|). \tag{10}$$

Working out the derivatives explicitly then gives the well-known result[32]

$$\vec{g}(\mathbf{r},\ \omega) = -\frac{4\pi}{k_o^2}\delta(\mathbf{r})\mathbf{e}_z\mathbf{e}_z + \frac{\mathrm{i}}{2\pi}\int \mathrm{d}^2\mathbf{k}_\parallel \frac{1}{\beta} \times \left(\vec{I} - \frac{1}{k_o^2}\mathbf{K}\mathbf{K}\right)\exp(\mathrm{i}\mathbf{K}\cdot\mathbf{r}), \tag{11}$$

where we have set

$$\mathbf{K} = \mathbf{k}_\parallel + \beta\,\mathrm{sgn}(z)\mathbf{e}_z. \tag{12}$$

It is interesting to note that a new delta function appears, but this one is not the same as the one in Eq. (7), representing the self-field. The delta function in Eq. (11) comes from differentiating $|z|$ twice with respect to z. Since the $\delta(\mathbf{r})$ part in Eq. (11) is not the self-field from Eq. (7), there should be an additional $\delta(\mathbf{r})$ contribution in the remaining integral over $\mathbf{k}_\parallel$. We shall show this explicitly in Section 7.

The angular spectrum representation [Eq. (11)] has a very transparent interpretation. Each partial wave has wave vector $\mathbf{K}$, and it follows from Eqs. (12) and (9) that $\mathbf{K}\cdot\mathbf{K} = k_o^2$. Therefore each partial wave has the same wave number k_o, and the corresponding plane wave is either traveling or evanescent, depending on the value of $k_\parallel$ compared with k_o. Also, when $\vec{g}(\mathbf{r},\ \omega)$ acts on the current density $\hat{\mathbf{j}}$, as in Eq. (2), then the electric field of the partial wave with wave vector $\mathbf{K}$ is proportional to $\hat{\mathbf{j}} - k_o^{-2}\mathbf{K}(\mathbf{K}\cdot\hat{\mathbf{j}})$, and, with $\mathbf{K}\cdot\mathbf{K} = k_o^2$, this shows $\mathbf{K}\cdot[\hat{\mathbf{j}} - k_o^{-2}\mathbf{K}(\mathbf{K}\cdot\hat{\mathbf{j}})] = 0$; e.g., the electric field of the partial wave is transverse.

4. TRAVELING AND EVANESCENT PARTS

Whether a plane wave $\exp(\mathrm{i}\mathbf{K}\cdot\mathbf{r})$ is traveling or evanescent depends on the z component of $\mathbf{K}$ and therefore on β. It follows from Eq. (9) that partial waves that have their vector $\mathbf{k}_\parallel$ inside the disk $0 \leqslant k_\parallel < k_o$ in the $\mathbf{k}_\parallel$ plane are traveling and waves with $k_\parallel > k_o$ are evanescent. Since this criterion depends only on $k_\parallel$, the magnitude of $\mathbf{k}_\parallel$, there is no reason for the present discussion to retain the dependence on the polar angle of vector $\mathbf{k}_\parallel$ in the $\mathbf{k}_\parallel$ plane. When polar coordinates $(k_\parallel, \bar{\phi})$ are used, the $\bar{\phi}$ dependence enters only through $\mathbf{K}$, and the integral over $\bar{\phi}$ can be performed. A great simplification in the notation follows by use of dimensionless variables. We shall use $1/k_o$ as the length scale, and we introduce $\alpha = k_\parallel/k_o$, $\rho = k_o r_\parallel$, $\zeta = k_o z$, $\hat{\beta} = \beta/k_o$, and $\vec{\gamma}(\mathbf{r},\ \omega) = \vec{g}(\mathbf{r},\ \omega)/k_o$. So α is the parameter that distinguishes between traveling waves $(0 \leqslant \alpha < 1)$ and evanescent waves $(\alpha > 1)$, and ρ and ζ are the dimensionless cylinder coordinates of the field point $\mathbf{r}$. The Green's tensor has an overall factor of k_o when everything else is expressed in dimensionless variables, so the introduction of $\vec{\gamma}$ also simplifies the notation.

Integration over $\bar{\phi}$ yields

$$\begin{aligned}\vec{\gamma}(\mathbf{r},\ \omega) = &-\frac{4\pi}{k_o^3}\delta(\mathbf{r})\mathbf{e}_z\mathbf{e}_z + \frac{1}{2}(\vec{I} + \mathbf{e}_z\mathbf{e}_z)M_0(\rho,\ \zeta) \\ &+ \frac{1}{2}\mathrm{sgn}(\zeta)(\hat{\mathbf{r}}_\parallel\mathbf{e}_z + \mathbf{e}_z\hat{\mathbf{r}}_\parallel)M_1(\rho,\ \zeta) \\ &+ \frac{1}{2}(\vec{I} - \mathbf{e}_z\mathbf{e}_z - 2\hat{\mathbf{r}}_\parallel\hat{\mathbf{r}}_\parallel)M_2(\rho,\ \zeta) \\ &+ \frac{1}{2}(\vec{I} - 3\mathbf{e}_z\mathbf{e}_z)N(\rho,\ \zeta).\end{aligned} \tag{13}$$

The dyadic parts of the Green's tensor are just combinations of $\vec{I}$, $\mathbf{e}_z$, and $\hat{\mathbf{r}}_\parallel$ (the radial unit vector in the xy plane). The dependence on the field point (ρ, ζ) enters through $\mathrm{sgn}(\zeta)$ and a set of auxiliary functions M_0, M_1, M_2, and N. These functions are defined by the following integral representations:

$$M_0(\rho,\ \zeta) = \mathrm{i}\int_0^\infty \mathrm{d}\alpha\frac{\alpha}{\hat{\beta}}J_0(\alpha\rho)\exp(\mathrm{i}\hat{\beta}|\zeta|), \tag{14}$$

$$M_1(\rho,\ \zeta) = 2\int_0^\infty \mathrm{d}\alpha\alpha^2 J_1(\alpha\rho)\exp(\mathrm{i}\hat{\beta}|\zeta|), \tag{15}$$

$$M_2(\rho,\ \zeta) = -\mathrm{i}\int_0^\infty \mathrm{d}\alpha\frac{\alpha^3}{\hat{\beta}}J_2(\alpha\rho)\exp(\mathrm{i}\hat{\beta}|\zeta|), \tag{16}$$

$$N(\rho,\ \zeta) = \mathrm{i}\int_0^\infty \mathrm{d}\alpha\alpha\hat{\beta}J_0(\alpha\rho)\exp(\mathrm{i}\hat{\beta}|\zeta|). \tag{17}$$

The ρ dependence enters through the Bessel functions $J_n(\alpha\rho)$, whereas all ζ dependence is contained in the factors $\exp(\mathrm{i}\hat{\beta}|\zeta|)$ in the integrands. For the time being, we shall assume $\zeta \neq 0$, since the last three integrals do not exist in the upper limit for $\zeta = 0$.

Since parameter α distinguishes between traveling and evanescent waves, we can readily identify both parts. For instance, the evanescent contribution to $M_0(\rho,\ \zeta)$ is

$$M_0(\rho,\ \zeta)^{\mathrm{ev}} = \int_1^\infty \mathrm{d}\alpha\frac{\alpha}{(\alpha^2 - 1)^{1/2}}J_0(\alpha\rho) \times \exp(-|\zeta|\sqrt{\alpha^2 - 1}), \tag{18}$$

and so on. In this way, the Green's tensor can be written as

$$\vec{\gamma}(\mathbf{r},\ \omega) = -\frac{4\pi}{k_o^3}\delta(\mathbf{r})\mathbf{e}_z\mathbf{e}_z + \vec{\gamma}(\mathbf{r},\ \omega)^{\mathrm{tr}} + \vec{\gamma}(\mathbf{r},\ \omega)^{\mathrm{ev}}, \tag{19}$$

and the traveling and evanescent parts follow when the integration range for α is limited to $[0, 1)$ and $(1, \infty)$, respectively, in Eqs. (14)–(17).

The integrals M_0 and M_2 have a factor $1/\hat{\beta}$ in their integrands, so these integrands have a singularity for $\alpha \rightarrow 1$. Since this is just on the borderline of $\alpha = 1$, these singularities appear in both the traveling and the evanescent parts. It is easy to see, however, that these singularities are integrable. For instance, if we set $\alpha^2 = 1 + u^2$ in Eq. (18), we find the representation

$$M_0(\rho,\ \zeta)^{\mathrm{ev}} = \int_0^\infty \mathrm{d}u J_0(\rho\sqrt{1+u^2})\exp(-u|\zeta|), \quad (20)$$

and the singularity has disappeared here. Similar transformations can be made for the other integrals with singularities. It is interesting to note that Eq. (20) has the form of a Laplace transform, so $M_0(\rho,\ \zeta)^{\mathrm{ev}}$ is the Laplace transform of $J_0[\rho(1+u^2)^{1/2}]$, with $|\zeta|$ as the Laplace parameter. The other three evanescent parts can also be written as Laplace transforms.

5. AUXILIARY FUNCTIONS

Explicit expressions for the integrals in Eqs. (14)–(17) can be found easily. The Green's tensor $\vec{\vec{g}}(\mathbf{r},\ \omega)$ is given by Eq. (7), with $\vec{\vec{d}}(\mathbf{r},\ \omega)$ from Eq. (6). Evaluating the derivatives in $\vec{\vec{d}}(\mathbf{r},\ \omega)$ and expressing the result in terms of the dimensionless radial distance q,

$$q = k_o r = \sqrt{\rho^2 + \zeta^2}, \quad (21)$$

then gives

$$\vec{\vec{\gamma}}(\mathbf{r},\ \omega) = -\frac{4\pi}{3k_o^3}\vec{\vec{I}}\delta(\mathbf{r}) + \vec{\vec{I}}\left(1 + \frac{\mathrm{i}}{q} - \frac{1}{q^2}\right)\frac{\exp(\mathrm{i}q)}{q} - \hat{\mathbf{r}}\hat{\mathbf{r}}\left(1 + \frac{3\mathrm{i}}{q} - \frac{3}{q^2}\right)\frac{\exp(\mathrm{i}q)}{q}, \quad (22)$$

where $\hat{\mathbf{r}} = \mathbf{r}/r$. This vector can be written in dimensionless cylinder coordinates as

$$\hat{\mathbf{r}} = \frac{1}{q}(\rho\hat{\mathbf{r}}_\parallel + \zeta\mathbf{e}_z). \quad (23)$$

Comparison of $\vec{\vec{\gamma}}(\mathbf{r},\ \omega)$ in Eq. (22) to the expression for $\vec{\vec{\gamma}}(\mathbf{r},\ \omega)$ in Eq. (13) then yields a set of four equations for the unknown integrals. Solving the set gives the result

$$M_0(\rho,\ \zeta) = \frac{\exp(\mathrm{i}q)}{q}, \quad (24)$$

$$M_1(\rho,\ \zeta) = -\frac{2\rho|\zeta|}{q^3}\left(1 + \frac{3\mathrm{i}}{q} - \frac{3}{q^2}\right)\exp(\mathrm{i}q), \quad (25)$$

$$M_2(\rho,\ \zeta) = \frac{\rho^2}{q^3}\left(1 + \frac{3\mathrm{i}}{q} - \frac{3}{q^2}\right)\exp(\mathrm{i}q), \quad (26)$$

$$N(\rho,\ \zeta) = -\frac{8\pi}{3k_o^3}\delta(\mathbf{r}) + \frac{1}{q^2}\left(\frac{1}{q} - \mathrm{i}\right)\exp(\mathrm{i}q) + \frac{\zeta^2}{q^3}\left(1 + \frac{3\mathrm{i}}{q} - \frac{3}{q^2}\right)\exp(\mathrm{i}q). \quad (27)$$

We recognize $M_0(\rho,\ \zeta)$ as the scalar Green's function:

$$M_0(\rho,\ \zeta) = g(\mathbf{r},\ \omega)/k_o, \quad (28)$$

and from Eqs. (25) and (26) we note that $M_2(\rho,\ \zeta)$ is related to $M_1(\rho,\ \zeta)$ as

$$M_2(\rho,\ \zeta) = -\frac{\rho}{2|\zeta|}M_1(\rho,\ \zeta). \quad (29)$$

Less obvious is the following relation for $N(\rho,\ \zeta)$:

$$N(\rho,\ \zeta) = -\frac{8\pi}{3k_o^3}\delta(\mathbf{r}) + \frac{1}{3}\left[M_0(\rho,\ \zeta) - \frac{|\zeta|}{\rho}M_1(\rho,\ \zeta) - M_2(\rho,\ \zeta)\right], \quad (30)$$

and here $M_2(\rho,\ \zeta)$ can be eliminated in favor of $M_1(\rho,\ \zeta)$ with Eq. (29). The appearance of the delta function in $N(\rho,\ \zeta)$ came from simply comparing the two representations of $\vec{\vec{\gamma}}(\mathbf{r},\ \omega)$. We shall show in Section 7 that the integral representation (17) for $N(\rho,\ \zeta)$ does indeed properly represent the self-field contribution to the angular spectrum.

6. EVANESCENT PART OF $M_2(\rho, \zeta)$

From Eqs. (29) and (30), it follows that all four integrals can be expressed in terms of $M_0(\rho,\ \zeta)$ and $M_1(\rho,\ \zeta)$ only. Since we are interested in the traveling and evanescent contributions to these integrals, the question arises whether these separate parts are also related in a simple way. In this section we show how Eq. (29) has to be modified if we consider the evanescent part only. The evanescent parts of $M_1(\rho,\ \zeta)$ and $M_2(\rho,\ \zeta)$ are defined by

$$M_1(\rho,\ \zeta)^{\mathrm{ev}} = 2\int_1^\infty \mathrm{d}\alpha\,\alpha^2 J_1(\alpha\rho)\exp(-|\zeta|\sqrt{\alpha^2-1}), \quad (31)$$

$$M_2(\rho,\ \zeta)^{\mathrm{ev}} = -\int_1^\infty \mathrm{d}\alpha\,\frac{\alpha^3}{(\alpha^2-1)^{1/2}}J_2(\alpha\rho) \times \exp(-|\zeta|\sqrt{\alpha^2-1}). \quad (32)$$

To relate both integrals, we integrate $M_2(\rho,\ \zeta)^{\mathrm{ev}}$ by parts, which gives

$$M_2(\rho,\ \zeta)^{\mathrm{ev}} = -\frac{1}{|\zeta|}J_2(\rho) - \frac{1}{|\zeta|}\int_1^\infty \mathrm{d}\alpha\exp(-|\zeta|\sqrt{\alpha^2-1}) \times \frac{\mathrm{d}}{\mathrm{d}\alpha}[\alpha^2 J_2(\alpha\rho)]. \quad (33)$$

The recursion relation for the Bessel functions $(\mathrm{d}/\mathrm{d}x)[x^2 J_2(x)] = x^2 J_1(x)$ then allows us to express the remaining integral in terms of $M_1(\rho,\ \zeta)^{\mathrm{ev}}$:

$$M_2(\rho,\ \zeta)^{\mathrm{ev}} = -\frac{1}{|\zeta|}J_2(\rho) - \frac{\rho}{2|\zeta|}M_1(\rho,\ \zeta)^{\mathrm{ev}}. \quad (34)$$

We note that, as compared with Eq. (29), an extra term, $-J_2(\rho)/|\zeta|$, appears.

The above derivation can be repeated for the traveling part, and in a similar way we then obtain

$$M_2(\rho,\ \zeta)^{\mathrm{tr}} = \frac{1}{|\zeta|}J_2(\rho) - \frac{\rho}{2|\zeta|}M_1(\rho,\ \zeta)^{\mathrm{tr}}. \quad (35)$$

Here again the term $J_2(\rho)/|\zeta|$ appears but with the opposite sign. When we add Eq. (35) to Eq. (34), we recover Eq. (29), as it should be. This shows that the traveling and evanescent parts of $M_2(\rho, \zeta)$ and $M_1(\rho, \zeta)$ are still related in a simple way, but, owing to the splitting, the traveling and evanescent parts acquire the additional term of $J_2(\rho)/|\zeta|$, with the opposite sign.

7. EVANESCENT PART OF $N(\rho, \zeta)$

In this section we shall show that Eq. (30) can also be broken up in an evanescent part and a traveling part. A complication here is the delta function, which should be properly represented by the integral representation of $N(\rho, \zeta)$. To this end, we recall the spectral representation of $\delta(\mathbf{r})$:

$$\delta(\mathbf{r}) = \frac{1}{(2\pi)^3}\int \mathrm{d}^3\mathbf{k}\exp(\mathrm{i}\mathbf{k}\cdot\mathbf{r}). \tag{36}$$

Then we use cylinder coordinates $(k_\parallel, \bar{\phi}, k_z)$ in $\mathbf{k}$ space and integrate over k_z and then over $\bar{\phi}$, just as in Section 4. This yields the formal representation

$$\delta(\mathbf{r}) = \frac{1}{2\pi}\delta(z)\int_0^\infty \mathrm{d}k_\parallel k_\parallel J_0(k_\parallel r_\parallel). \tag{37}$$

Then we transform again to dimensionless variables, and we keep the upper limit finite:

$$\delta(\mathbf{r}) = \frac{k_o^3}{2\pi}\delta(\zeta)\int_0^A \mathrm{d}\alpha\,\alpha J_0(\alpha\rho), \qquad A \to \infty. \tag{38}$$

With a recursion relation for the Bessel functions, this integral can be evaluated, resulting in the representation[33]

$$\delta(\mathbf{r}) = \frac{k_o^3}{2\pi}\delta(\zeta)\frac{A}{\rho}J_1(A\rho), \qquad A \to \infty. \tag{39}$$

The evanescent part of $N(\rho, \zeta)$ is

$$N(\rho, \zeta)^{\mathrm{ev}} = -\int_1^\infty \mathrm{d}\alpha\,\alpha\sqrt{\alpha^2 - 1}J_0(\alpha\rho) \times \exp(-|\zeta|\sqrt{\alpha^2 - 1}). \tag{40}$$

When we use the recursion relation $xJ_0(x) = (\mathrm{d}/\mathrm{d}x)[xJ_1(x)]$, keep the upper limit finite, say A, and integrate by parts, then $N(\rho, \zeta)^{\mathrm{ev}}$ becomes

$$N(\rho, \zeta)^{\mathrm{ev}} = -\frac{A}{\rho}J_1(A\rho)\sqrt{A^2 - 1}\exp(-|\zeta|\sqrt{A^2 - 1}) + \frac{1}{\rho}\int_1^A \mathrm{d}\alpha\,\alpha J_1(\alpha\rho)\frac{\mathrm{d}}{\mathrm{d}\alpha}\left[\sqrt{\alpha^2 - 1} \times \exp(-|\zeta|\sqrt{\alpha^2 - 1})\right], \qquad A \to \infty. \tag{41}$$

With the familiar representation of the one-dimensional delta function

$$\delta(\zeta) = \tfrac{1}{2}\sqrt{A^2 - 1}\exp(-|\zeta|\sqrt{A^2 - 1}), \qquad A \to \infty, \tag{42}$$

and Eq. (39), we see that the first term on the right-hand side of Eq. (41) represents $\delta(\mathbf{r})$, apart from a constant. Then we work out the derivative $\mathrm{d}/\mathrm{d}\alpha$ and use Eq. (31), which yields

$$N(\rho, \zeta)^{\mathrm{ev}} = -\frac{4\pi}{k_o^3}\delta(\mathbf{r}) - \frac{|\zeta|}{2\rho}M_1(\rho, \zeta)^{\mathrm{ev}} + \frac{1}{\rho}\int_1^\infty \mathrm{d}\alpha\frac{\alpha^2}{(\alpha^2 - 1)^{1/2}}J_1(\alpha\rho) \times \exp(-|\zeta|\sqrt{\alpha^2 - 1}). \tag{43}$$

Comparison with Eq. (30) shows some similarity with $N(\rho, \zeta)$, but the delta function does not have the correct overall factor. Apparently, the remaining integral in Eq. (43) must still have a hidden delta function.

Next, we eliminate $J_2(\alpha\rho)$ with a recursion relation from representation (32) for $M_2(\rho, \zeta)^{\mathrm{ev}}$, in favor of $J_0(\alpha\rho)$ and $J_1(\alpha\rho)$. After some rearrangements, we then obtain

$$M_2(\rho, \zeta)^{\mathrm{ev}} = M_0(\rho, \zeta)^{\mathrm{ev}} - N(\rho, \zeta)^{\mathrm{ev}} - \frac{2}{\rho}\int_1^\infty \mathrm{d}\alpha\frac{\alpha^2}{(\alpha^2 - 1)^{1/2}}J_1(\alpha\rho) \times \exp(-|\zeta|\sqrt{\alpha^2 - 1}). \tag{44}$$

Then we note that both Eqs. (43) and (44) contain the same integral, and both involve $N(\rho, \zeta)^{\mathrm{ev}}$. We eliminate the integral in favor of $N(\rho, \zeta)^{\mathrm{ev}}$, which then finally gives

$$N(\rho, \zeta)^{\mathrm{ev}} = -\frac{8\pi}{3k_o^3}\delta(\mathbf{r}) + \frac{1}{3}\left[M_0(\rho, \zeta)^{\mathrm{ev}} - \frac{|\zeta|}{\rho}M_1(\rho, \zeta)^{\mathrm{ev}} - M_2(\rho, \zeta)^{\mathrm{ev}}\right]. \tag{45}$$

This result is identical in form as Eq. (30) for the unsplit $N(\rho, \zeta)$. This shows explicitly that the integral representation for $N(\rho, \zeta)$ does indeed represent the self-field properly, and it also proves that this delta function contribution is entirely included in the evanescent part. Furthermore, no extra term appears because of the splitting, as was the case in the previous Section 6 with the splitting of $M_2(\rho, \zeta)$.

The computation of this section can be repeated for the integral representing the traveling part of $N(\rho, \zeta)$. The only difference is then that the integrated part in Eq. (41), which provided the delta function, is now identically zero. Therefore the result for the traveling part is

$$N(\rho, \zeta)^{\mathrm{tr}} = \frac{1}{3}\left[M_0(\rho, \zeta)^{\mathrm{tr}} - \frac{|\zeta|}{\rho}M_1(\rho, \zeta)^{\mathrm{tr}} - M_2(\rho, \zeta)^{\mathrm{tr}}\right], \tag{46}$$

and, when added to Eq. (45), the result reproduces Eq. (30), which was derived directly from the Green's tensor.

8. ASYMPTOTIC EXPANSION OF THE EVANESCENT PART

To obtain asymptotic expansions of the evanescent parts of the auxiliary functions, we could first change variables according to $\alpha^2 = 1 + u^2$ in the integral representations.

Then each integral becomes of the Laplace type, as in Eq. (20), and asymptotic expansions can be obtained by standard methods.[34] There is, however, an interesting step that can be made first, which facilitates the computation considerably. In Section 4 it was shown that the transformation $\alpha^2 = 1 + u^2$ removes the singularities from the integrands. An alternative transformation follows from the identity

$$\int_1^\infty \mathrm{d}\alpha \frac{\alpha^{n+1}}{(\alpha^2-1)^{1/2}} J_n(\alpha\rho)\exp(-|\zeta|\sqrt{\alpha^2-1}) = \frac{1}{|\zeta|}J_n(\rho) + \frac{\rho}{|\zeta|}\int_1^\infty \mathrm{d}\alpha\,\alpha^n J_{n-1}(\alpha\rho) \times \exp(-|\zeta|\sqrt{\alpha^2-1}), \quad n = 0, 1, 2, \ldots, \tag{47}$$

which can be derived from integration by parts and a recursion relation for the Bessel functions. We note that $M_0(\rho, \zeta)^{\mathrm{ev}}$, Eq. (18), and $M_2(\rho, \zeta)^{\mathrm{ev}}$, Eq. (32), are of this type. For $n = 0$ we have $J_{-1}(\alpha\rho) = -J_1(\alpha\rho)$, and this becomes

$$M_0(\rho, \zeta)^{\mathrm{ev}} = \frac{1}{|\zeta|}J_0(\rho) - \frac{\rho}{|\zeta|}\int_1^\infty \mathrm{d}\alpha J_1(\alpha\rho) \times \exp(-|\zeta|\sqrt{\alpha^2-1}). \tag{48}$$

Now we make the substitution $\alpha^2 = 1 + u^2$, which gives

$$M_0(\rho, \zeta)^{\mathrm{ev}} = \frac{1}{|\zeta|}J_0(\rho) - \frac{\rho}{|\zeta|}\int_0^\infty \mathrm{d}u \times \exp(-u|\zeta|)\frac{u}{(1+u^2)^{1/2}} J_1(\rho\sqrt{1+u^2}). \tag{49}$$

If we then integrate by parts, the integrated part is zero. Integrating twice yields

$$M_0(\rho, \zeta)^{\mathrm{ev}} = \frac{1}{|\zeta|}J_0(\rho) - \frac{\rho}{|\zeta|^3}J_1(\rho) - \frac{\rho}{|\zeta|^3}\int_0^\infty \mathrm{d}u\,\exp(-u|\zeta|) \times \frac{\mathrm{d}^2}{\mathrm{d}u^2}\frac{u}{(1+u^2)^{1/2}}J_1(\rho\sqrt{1+u^2}), \tag{50}$$

as an exact result. It is now clear that in this fashion we arrive at an asymptotic expansion with $|\zeta|$ as the large parameter and with the value of ρ fixed. The next integrated part vanishes again, and therefore the next contribution is $\mathcal{O}(|\zeta|^{-5})$. This gives

$$M_0(\rho, \zeta)^{\mathrm{ev}} = \frac{1}{|\zeta|}J_0(\rho) - \frac{\rho}{|\zeta|^3}J_1(\rho) + \mathcal{O}(|\zeta|^{-5}), \tag{51}$$

which is the asymptotic expansion of $M_0(\rho, \zeta)^{\mathrm{ev}}$.

The asymptotic expansion of $M_1(\rho, \zeta)^{\mathrm{ev}}$ can be derived in a similar way from Eq. (31) with the result

$$M_1(\rho, \zeta)^{\mathrm{ev}} = \frac{2}{|\zeta|^2}J_1(\rho) + \frac{6\rho}{|\zeta|^4}J_0(\rho) + \mathcal{O}(|\zeta|^{-6}). \tag{52}$$

The remaining two integrals can be handled in the same way, but with Eqs. (34) and (45) we find immediately

$$M_2(\rho, \zeta)^{\mathrm{ev}} = -\frac{1}{|\zeta|}J_2(\rho) - \frac{\rho}{|\zeta|^3}J_1(\rho) + \mathcal{O}(|\zeta|^{-5}), \tag{53}$$

$$N(\rho, \zeta)^{\mathrm{ev}} = -\frac{2}{|\zeta|^3}J_0(\rho) + \mathcal{O}(|\zeta|^{-5}). \tag{54}$$

The asymptotic expansion of the auxiliary functions can be substituted into Eq. (13) to obtain the expansion of the evanescent part of the Green's tensor $\tilde{\gamma}(\mathbf{r}, \omega)^{\mathrm{ev}}$. Figure 1 illustrates the accuracy of the asymptotic approximation for the case of $M_0(\rho, \zeta)^{\mathrm{ev}}$ with $\rho = 5$. Shown is the exact result, the first approximation, and the approximation up to $\mathcal{O}(|\zeta|^{-3})$. The exact result was obtained by numerical integration. At this point it should be noted that for numerical purposes the transformation to Eq. (20) is useful. Although the singularity $(\alpha^2 - 1)^{-1/2}$ is integrable, it is not attractive from a numerical point of view. Figure 1 shows that the approximation sets in at approximately one wavelength distance ($\zeta = 2\pi$) from the xy plane. We have verified numerically that the approximations of Eqs. (52)–(54) are equally accurate.

When the field point is on the z axis, we have $\rho = 0$, and the integrals in Eqs. (18), (31), (32), and (40) can be evaluated explicitly. We find

$$M_0(0, \zeta)^{\mathrm{ev}} = \frac{1}{|\zeta|}, \tag{55}$$

$$M_1(0, \zeta)^{\mathrm{ev}} = M_2(0, \zeta)^{\mathrm{ev}} = 0, \tag{56}$$

$$N(0, \zeta)^{\mathrm{ev}} = -\frac{2}{|\zeta|^3}, \tag{57}$$

for $\zeta \neq 0$. However, when we set $\rho = 0$ in the asymptotic expansion [Eqs. (51)–(54)], we obtain precisely the same result, provided that we set $\mathcal{O}(|\zeta|^{-5})$ and $\mathcal{O}(|\zeta|^{-6})$ equal to zero. Therefore the asymptotic expansion with the terms shown explicitly in Eqs. (51)–(54) is exact on the z axis for

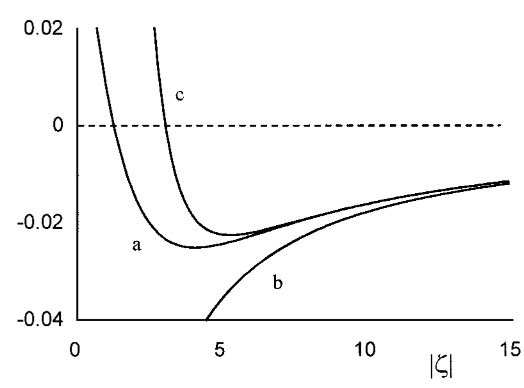

Fig. 1. Evanescent part of the auxiliary function $M_0(\rho, \zeta)$ (curve a), as a function of $|\zeta|$ for $\rho = 5$. Curve b is the asymptotic approximation $J_0(\rho)/|\zeta|$, and curve c is the asymptotic approximation with both terms from Eq. (51). The exact value remains finite for $|\zeta| \to 0$, but the approximations diverge near the xy plane.

all ζ, $\zeta \neq 0$. The evanescent part of the Green's tensor for a point on the z axis ($\zeta \neq 0$) is then found to be

$$\overleftrightarrow{\gamma}(\mathbf{r},\ \omega)^{\mathrm{ev}} = \frac{1}{2|\zeta|}(\overleftrightarrow{I} + \mathbf{e}_z\mathbf{e}_z) - \frac{1}{|\zeta|^3}(\overleftrightarrow{I} - 3\mathbf{e}_z\mathbf{e}_z), \quad (58)$$

in agreement with Ref. 17.

Furthermore, if the normal distance between the field point and the z axis is less than approximately a fraction of a wavelength, we have $J_0(\rho) \approx 1$ and $J_n(\rho) \approx 0$, $n = 1, 2, \ldots$, and we find to leading order in $|\zeta|$

$$\overleftrightarrow{\gamma}(\mathbf{r},\ \omega)^{\mathrm{ev}} \approx \frac{1}{2|\zeta|}(\overleftrightarrow{I} + \mathbf{e}_z\mathbf{e}_z). \quad (59)$$

In this cylindrical region around the z axis we have $|\zeta| \approx q$, and therefore near the z axis the evanescent waves survive as $\mathcal{O}(q^{-1})$ in the far field. This result generalizes conclusions by others[17,18,23] that the evanescent waves reach the far field as $\mathcal{O}(q^{-1})$ on the z axis. It was shown recently[35] for the case of the scalar wave that the evanescent waves do reach the far field in a "forward needle" of finite width, in agreement with our results.

It is more common to consider the asymptotic approximation as a function of the radial distance $q = k_o r$ to the source, for a fixed polar angle θ. We then have $\rho = q \sin\theta$ and $\zeta = q\cos\theta$. For $\theta = 0$ or $\theta = \pi$, this gives $\rho = 0$, $q = |\zeta|$, and the approximation of the Green's tensor for the evanescent part is given by approximation (59). We shall now assume $\theta \neq 0, \pi$. Then when q becomes large, ρ also becomes large, and we can use the asymptotic approximation of the Bessel functions:

$$J_n(\rho) \approx \left(\frac{2}{\pi\rho}\right)^{1/2} \cos\left(\rho - \frac{1}{2}n\pi - \frac{1}{4}\pi\right). \quad (60)$$

Seen as a function of q, all Bessel functions are $\mathcal{O}(q^{-1/2})$, and both ρ and ζ are $\mathcal{O}(q)$. To lowest order in $1/q$, the first term of $M_0(\rho, \zeta)^{\mathrm{ev}}$, Eq. (51), and the first term of $M_2(\rho, \zeta)^{\mathrm{ev}}$, Eq. (53), contribute. With $J_2(\rho) \approx -J_0(\rho)$ we have $M_2(\rho, \zeta)^{\mathrm{ev}} \approx M_0(\rho, \zeta)^{\mathrm{ev}}$, and we obtain for the evanescent part of the Green's tensor:

$$\overleftrightarrow{\gamma}(\mathbf{r},\ \omega)^{\mathrm{ev}} \approx \frac{1}{q^{3/2}}(\overleftrightarrow{I} - \hat{\mathbf{r}}_\parallel\hat{\mathbf{r}}_\parallel)\frac{1}{|\cos\theta|}\left(\frac{2}{\pi\sin\theta}\right)^{1/2} \times \cos\left(\frac{\pi}{4} - q\sin\theta\right), \quad (61)$$

for $\rho = q\sin\theta$ sufficiently large. This shows the typical $\mathcal{O}(q^{-3/2})$ behavior for field points far away from the z axis. Approximation (61) is in agreement with Ref. 18, although it should be noted that there the dyadic term $\hat{\mathbf{r}}_\parallel\hat{\mathbf{r}}_\parallel$ is missing. It is interesting to note that near the z axis, approximation (59), and far away from the z axis, approximation (61), the dyadic parts are also different. Finally, near the xy plane approximation (61) is obviously invalid, since it was derived from the expansion for $|\zeta|$ large.

9. ANALYTIC SOLUTION

It appears possible to evaluate the integrals for the evanescent parts of the auxiliary functions in closed form. The integral in Eq. (18) can be found from a tabulated integral,[36] and this expression has been used to study diffraction problems involving the scalar Green's function.[37,38] The result is

$$M_0(\rho, \zeta)^{\mathrm{ev}} = \frac{1}{q}[2U_0(q - |\zeta|, \rho) - J_0(\rho)], \quad (62)$$

in terms of a Lommel function U_0. These functions of two variables are defined by[39,40]

$$U_\ell(a, b) = \sum_{m=0}^{\infty}(-1)^m\left(\frac{a}{b}\right)^{\ell+2m} J_{\ell+2m}(b), \quad (63)$$

for ℓ integer and $b > a > 0$. We shall need these functions for $\ell = 0, 1$, $a = q - |\zeta|$, and $b = \rho$. With the relations between ρ, ζ, and q, we then find that $a/b = \tan(\theta/2)$ for $0 \leqslant \theta \leqslant \pi/2$ and $a/b = \cot(\theta/2)$ for $\pi/2 \leqslant \theta \leqslant \pi$, showing that the Lommel functions are basically functions of θ and ρ.

From Eqs. (18) and (31) we derive that $M_1(\rho, \zeta)^{\mathrm{ev}}$ can be found from $M_0(\rho, \zeta)^{\mathrm{ev}}$ by differentiation:

$$M_1(\rho, \zeta)^{\mathrm{ev}} = 2\,\mathrm{sgn}(\zeta)\frac{\partial^2}{\partial\rho\partial\zeta}M_0(\rho, \zeta)^{\mathrm{ev}}. \quad (64)$$

Apparently, we need derivatives of the Lommel functions $U_\ell(a, b)$ with respect to ρ and ζ. With the known derivatives with respect to a and b[39], these derivatives are found to be

$$\frac{\partial U_\ell}{\partial\rho} = \frac{\rho}{q}\left[\frac{1}{2}\left(\frac{a}{b}\right)^{\ell-1} J_{\ell-1} - U_{\ell+1}\right], \quad (65)$$

$$\frac{\partial U_\ell}{\partial\zeta} = -\frac{\zeta}{q}U_{\ell+1} - \frac{a}{2q}\mathrm{sgn}(\zeta)\left(\frac{a}{b}\right)^{\ell-1} J_{\ell-1}. \quad (66)$$

Here and in the remainder of this section we suppress the arguments of the Bessel functions (ρ) and the Lommel functions (a, b). Furthermore, the Lommel functions obey the recursion relation

$$U_\ell + U_{\ell+2} = \left(\frac{a}{b}\right)^{\ell} J_\ell, \quad (67)$$

which allows us to express all Lommel functions in terms of U_0 and U_1 only. Similarly, in this section we shall express all Bessel functions in terms of J_0 and J_1. We then obtain the result for $M_1(\rho, \zeta)^{\mathrm{ev}}$:

$$M_1(\rho, \zeta)^{\mathrm{ev}} = \frac{2}{q^2}\left\{\rho J_0 + \frac{\rho|\zeta|}{q}(J_0 - 2U_0)\left(1 - \frac{3}{q^2}\right) + \frac{1}{q^2}[6\rho|\zeta|U_1 + (\zeta^2 - 2\rho^2)J_1]\right\}. \quad (68)$$

From Eqs. (18) and (32) we derive

$$M_2(\rho, \zeta)^{\mathrm{ev}} = \left(\frac{1}{\rho} - \frac{\partial}{\partial\rho}\right)\frac{\partial}{\partial\rho}M_0(\rho, \zeta)^{\mathrm{ev}}, \quad (69)$$

and therefore we can obtain also $M_2(\rho, \zeta)^{\mathrm{ev}}$ by differentiation. Alternatively, we can use Eq. (34), which yields immediately

$$M_2(\rho, \zeta)^{\mathrm{ev}} = \frac{1}{q^2}\left[|\zeta|J_0 - |\zeta|J_1\left(\frac{2}{\rho} + \frac{3\rho}{q^2}\right) - \frac{6\rho^2}{q^2}U_1 - \frac{\rho^2}{q}(J_0 - 2U_0)\left(1 - \frac{3}{q^2}\right)\right], \tag{70}$$

and we have verified that both approaches give the same result. Finally, from Eq. (45) we find the evanescent part of $N(\rho, \zeta)$ to be

$$N(\rho, \zeta)^{\mathrm{ev}} = -\frac{8\pi}{3k_o^3}\delta(\mathbf{r}) + \frac{1}{q^3}\left[-2\zeta^2U_0 - |\zeta|(q + |\zeta|)J_0 + \frac{3\rho|\zeta|}{q}J_1 + (2qU_1 + 2U_0 - J_0) \times \left(1 - \frac{3\zeta^2}{q^2}\right)\right]. \tag{71}$$

We have verified the expressions above by comparing them with results obtained with numerical integration.

Since the evanescent parts are pure real, it follows from Eq. (19) that the imaginary part of the Green's tensor resides entirely in the traveling part. We therefore have

$$\mathrm{Im}[\overleftrightarrow{\gamma}(\mathbf{r}, \omega)^{\mathrm{tr}}] = \mathrm{Im}[\overleftrightarrow{\gamma}(\mathbf{r}, \omega)], \tag{72}$$

and the right-hand side can be found by taking the imaginary part of the right-hand side of Eq. (22). For the corresponding auxiliary functions we then find

$$\mathrm{Im}[M_0(\rho, \zeta)^{\mathrm{tr}}] = \frac{\sin q}{q}, \tag{73}$$

$$\mathrm{Im}[M_1(\rho, \zeta)^{\mathrm{tr}}] = -\frac{2\rho|\zeta|}{q^3}\left[\frac{3\cos q}{q} + \left(1 - \frac{3}{q^2}\right)\sin q\right], \tag{74}$$

$$\mathrm{Im}[M_2(\rho, \zeta)^{\mathrm{tr}}] = \frac{\rho^2}{q^3}\left[\frac{3\cos q}{q} + \left(1 - \frac{3}{q^2}\right)\sin q\right], \tag{75}$$

$$\mathrm{Im}[N(\rho, \zeta)^{\mathrm{tr}}] = \frac{\zeta^2}{q^3}\sin q + \frac{1}{q^2}\left(1 - \frac{3\zeta^2}{q^2}\right) \times \left(\frac{\sin q}{q} - \cos q\right). \tag{76}$$

Since only the real part of $\overleftrightarrow{\gamma}(\mathbf{r}, \omega)$ splits into traveling and evanescent, the real part of the traveling part follows from

$$\mathrm{Re}[\overleftrightarrow{\gamma}(\mathbf{r}, \omega)^{\mathrm{tr}}] = \mathrm{Re}[\overleftrightarrow{\gamma}(\mathbf{r}, \omega)] - \overleftrightarrow{\gamma}(\mathbf{r}, \omega)^{\mathrm{ev}}, \tag{77}$$

and the auxiliary functions follow accordingly. The real part of $\overleftrightarrow{\gamma}(\mathbf{r}, \omega)$ can be found from Eqs. (24)–(27), and for the evanescent part we use Eqs. (62), (68), (70), and (71).

For a field point on the z axis we have $U_0 = 1$ and $U_1 = 0$, and it follows that the solutions above reduce to Eqs. (55)–(57). The corresponding traveling parts are

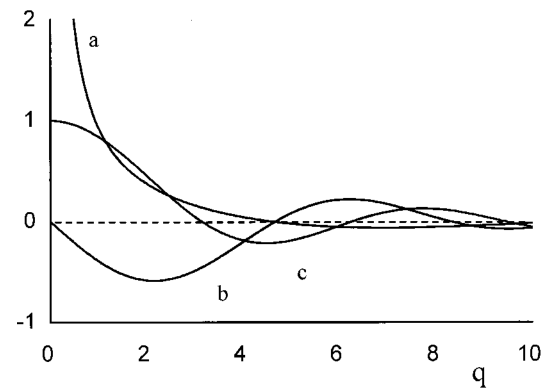

Fig. 2. Illustration of the splitting of the function $M_0(\rho, \zeta)$ into its evanescent part (curve a) and traveling part. The real and imaginary parts of the traveling part are shown as curves b and c, respectively. The value of the traveling part at the origin is equal to i, whereas the evanescent part diverges.

$$M_0(0, \zeta)^{\mathrm{tr}} = -\frac{1}{|\zeta|}[1 - \exp(\mathrm{i}|\zeta|)], \tag{78}$$

$$M_1(0, \zeta)^{\mathrm{tr}} = M_2(0, \zeta)^{\mathrm{tr}} = 0, \tag{79}$$

$$N(0, \zeta)^{\mathrm{tr}} = \frac{2}{|\zeta|^3}[1 - \exp(\mathrm{i}|\zeta|)] + \left(1 + \frac{2\mathrm{i}}{|\zeta|}\right)\frac{\exp(\mathrm{i}|\zeta|)}{|\zeta|}. \tag{80}$$

It is particularly interesting to consider the limit $|\zeta| \to 0$ along the z axis, for which we find

$$M_0(0, 0)^{\mathrm{tr}} = \mathrm{i}, \tag{81}$$

$$N(0, 0)^{\mathrm{tr}} = \mathrm{i}/3. \tag{82}$$

The corresponding Green's tensor is

$$\overleftrightarrow{\gamma}(0)^{\mathrm{tr}} = \frac{2\mathrm{i}}{3}\overleftrightarrow{I}, \tag{83}$$

showing that the traveling part remains finite at the origin, and therefore all singular behavior is accounted for by evanescent waves. Equation (83) agrees with Ref. 17. Figure 2 illustrates a typical splitting into three parts (evanescent, traveling real, and traveling imaginary), shown as a function of q. We see from the graph that indeed $M_0(0, 0)^{\mathrm{tr}} = \mathrm{i}$.

10. SOLUTION IN THE *XY* PLANE

As was mentioned in Section 4, the integrals representing $M_1(\rho, \zeta)$, $M_2(\rho, \zeta)$, and $N(\rho, \zeta)$ diverge for $\zeta = 0$. Nevertheless, they represent the functions given by Eqs. (25)–(27), which do exist for $\zeta = 0$, and are finite (let $\rho \neq 0$ in this section). The problem obviously lies in the evanescent part, but with the results from Section 9 we can now consider the limit $|\zeta| \to 0$. The first argument of the Lommel functions becomes $q - |\zeta| \to \rho$, so we can use the known special cases[35]

$$U_0(\rho, \rho) = \tfrac{1}{2}[J_0(\rho) + \cos\rho], \tag{84}$$

$$U_1(\rho, \rho) = \tfrac{1}{2} \sin \rho. \tag{85}$$

We then obtain from Section 9 the limiting values for the evanescent parts:

$$M_0(\rho, 0)^{\mathrm{ev}} = \frac{\cos \rho}{\rho}, \tag{86}$$

$$M_1(\rho, 0)^{\mathrm{ev}} = -\frac{2}{\rho} J_2(\rho), \tag{87}$$

$$M_2(\rho, 0)^{\mathrm{ev}} = \frac{\cos \rho}{\rho} - \frac{3}{\rho^2}\left(\sin \rho + \frac{\cos \rho}{\rho}\right), \tag{88}$$

$$N(\rho, 0)^{\mathrm{ev}} = \frac{1}{\rho^2}\left(\sin \rho + \frac{\cos \rho}{\rho}\right). \tag{89}$$

We find that all functions exist for $|\zeta| \to 0$, and therefore the fact that the integrals do not formally exist for $\zeta = 0$ should be considered a mathematical artifact. The exception here is, of course, the representation of $N(\rho, 0)^{\mathrm{ev}}$, since this divergence partially represents the self-field. However, if we split off this self-field, as in Eq. (45), then the remaining part is finite for $|\zeta| \to 0$ and given by Eq. (89).

The imaginary parts of the traveling parts in the xy plane follow from Eqs. (73)–(76), with $\zeta = 0$ and $q = \rho$. The real parts of the traveling parts can be found by taking the real parts of Eqs. (24)–(27) and subtracting the evanescent parts, given above. We then find

$$\mathrm{Re}[M_0(\rho, 0)^{\mathrm{tr}}] = \mathrm{Re}[M_2(\rho, 0)^{\mathrm{tr}}] = \mathrm{Re}[N(\rho, 0)^{\mathrm{tr}}] = 0, \tag{90}$$

$$\mathrm{Re}[M_1(\rho, 0)^{\mathrm{tr}}] = \frac{2}{\rho} J_2(\rho). \tag{91}$$

In the xy plane, the real part of the Green's tensor consists entirely of evanescent waves, except for the M_1 component. Since we also have $\mathrm{Re}[M_1(\rho, 0)] = 0$, we reach the peculiar conclusion that a component that is identically zero splits into two equal and opposite parts, given by Eqs. (87) and (91). This situation is reminiscent of the appearance of the terms $\pm J_2(\rho)/|\zeta|$ in the splitting of $M_2(\rho, \zeta)$ in Section 6.

With the solution given above, it is easy to verify that in the limit $\rho \to 0$ the traveling part reduces again to Eq. (83), as it should. For ρ large, the dominant contribution is $\mathcal{O}(\rho^{-1})$, and we have

$$M_0(\rho, 0) = \frac{\exp(i\rho)}{\rho} \approx M_2(\rho, 0), \tag{92}$$

with all other functions of higher order. The real parts, $(\cos \rho)/\rho$, are entirely evanescent, whereas the imaginary parts, $(\sin \rho)/\rho$, are entirely traveling. The corresponding Green's tensor is

$$\overleftrightarrow{\gamma}(\mathbf{r}, \omega)_{\zeta=0} \approx (\overleftrightarrow{I} - \hat{\mathbf{r}}_{\parallel}\hat{\mathbf{r}}_{\parallel}) \frac{\exp(i\rho)}{\rho}, \qquad \rho \text{ large}. \tag{93}$$

11. TRAVELING WAVES

So far we have mainly focused on the properties of the evanescent waves by means of the angular spectrum expansion. In this section we shall show that the traveling waves have their origin directly in the field itself and can be obtained without reference to the angular spectrum. To this end, we note that the integrals in Eqs. (14)–(16) have the form

$$M_n(\rho, \zeta) = \int_0^\infty \mathrm{d}\alpha \alpha \{\ldots\} J_n(\alpha\rho), \tag{94}$$

with $\{\ldots\}$ a function of α and ζ but not of ρ. The corresponding traveling part is then

$$M_n(\rho, \zeta)^{\mathrm{tr}} = \int_0^1 \mathrm{d}\alpha \alpha \{\ldots\} J_n(\alpha\rho). \tag{95}$$

We note that Eq. (94) has the form of a Fourier–Bessel transform, which can be inverted according to[41]

$$\{\ldots\} = \int_0^\infty \mathrm{d}\rho' \rho' M_n(\rho', \zeta) J_n(\alpha\rho'). \tag{96}$$

Then we substitute this expression into the right-hand side of Eq. (95) and rearrange the terms. Then the traveling part can be written as

$$M_n(\rho, \zeta)^{\mathrm{tr}} = \int_0^\infty \mathrm{d}\rho' F_n(\rho, \rho') M_n(\rho', \zeta), \qquad n = 0, 1, 2, \tag{97}$$

with $F_n(\rho, \rho')$ universal functions, defined by

$$F_n(\rho, \rho') = \rho' \int_0^1 \mathrm{d}\alpha \alpha J_n(\alpha\rho) J_n(\alpha\rho'), \qquad n = 0, 1, \ldots. \tag{98}$$

Equation (97) shows that the traveling part of $M_n(\rho, \zeta)$ can be obtained directly from $M_n(\rho, \zeta)$ by means of a filter-type operation with a field-independent filter function $F_n(\rho, \rho')$.

The integral in Eq. (98) can be evaluated by quadrature, and the result is

$$F_n(\rho, \rho') = \frac{\rho'}{\rho^2 - (\rho')^2}[\rho J_n(\rho') J_{n+1}(\rho) - \rho' J_n(\rho) J_{n+1}(\rho')]. \tag{99}$$

For $\rho' = \rho$ we take the limit $\rho' \to \rho$, which yields

$$F_n(\rho, \rho) = \frac{1}{2}\rho\left\{[J_n(\rho)']^2 + \left(1 - \frac{n^2}{\rho^2}\right) J_n(\rho)^2\right\}, \tag{100}$$

with $J_n(\rho)'$ the derivative of $J_n(\rho)$. Figure 3 shows the behavior of $F_1(\rho, \rho')$ as a function of ρ' for a fixed ρ. It appears that $F_n(\rho, \rho')$ has a strong maximum at $\rho' \approx \rho$ for ρ sufficiently large. Indeed, if we use the asymptotic form Eq. (60) of the Bessel functions and take $\rho' \approx \rho$, then Eq. (99) reduces to

$$F_n(\rho, \rho') \approx \frac{1}{\pi} \frac{\sin(\rho - \rho')}{\rho - \rho'}, \tag{101}$$

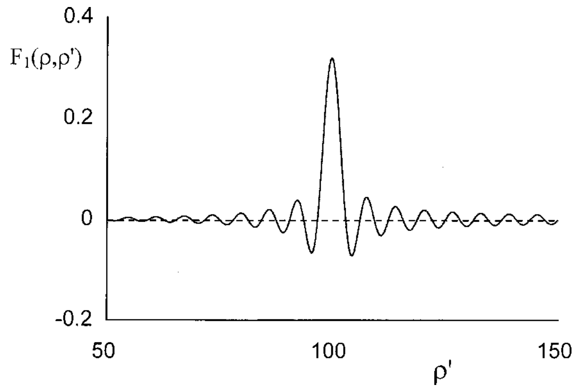

Fig. 3. This graph shows the filter function $F_1(\rho, \rho')$ as a function of ρ', for $\rho = 100$. The pronounced peak is located near $\rho' = 100$, and the peak height is approximately $1/\pi$, as indicated by Eq. (101).

e.g., a sinc function around $\rho' = \rho$. This illustrates that the traveling part of the field, as determined by Eq. (97), acquires its dominant contribution from the field in the neighborhood of the field point (ρ, ζ).

As shown in Section 9, only the real part of the auxiliary functions is affected by the splitting into a traveling and an evanescent part. The imaginary part ends up entirely in the traveling part, as indicated by Eq. (72). Since the filter functions are real, the imaginary part of Eq. (97) becomes

$$\mathrm{Im}[M_n(\rho, \zeta)] = \int_0^\infty \mathrm{d}\rho' F_n(\rho, \rho')\mathrm{Im}[M_n(\rho', \zeta)]. \tag{102}$$

This exhibits the remarkable feature that the functions $\mathrm{Im}[M_n(\rho, \zeta)]$, given by Eqs. (73)–(75), pass through the filter $F_n(\rho, \rho')$ undistorted.

12. CONCLUSIONS

We have studied the traveling and evanescent parts of the electromagnetic Green's tensor $\vec{g}(\mathbf{r}, \omega) = k_o \vec{\gamma}(\mathbf{r}, \omega)$. We have shown that the Green's tensor can be split with the help of four auxiliary functions on the basis of an angular spectrum representation. The self-field was shown to be properly represented as a part of the function $N(\rho, \zeta)^{\mathrm{ev}}$ and could be separated from it, as shown in Eq. (45). We have derived an asymptotic expansion for the evanescent part of the Green's tensor as a function of $|\zeta|$ and have also obtained a closed-form solution for the evanescent part in terms of the Lommel functions. This analytic solution appeared to be particularly useful for the study of the Green's tensor in the neighborhood of the xy plane, where the asymptotic expansion is invalid. As for the traveling waves, we have shown in Section 11 that the traveling part of the Green's tensor can be found without direct reference to its angular spectrum representation. Each auxiliary function can be filtered with a filter function $F_n(\rho, \rho')$, which then immediately yields the traveling component of that function.

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