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Atomic response to the Lorentz wave

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Abstract. In this paper the fluorescence radiation, emitted by a two-level atom in a perturber bath, is considered. The atom is driven by a single-mode laser with a Lorentzian profile, which is brought about by a fluctuating phase. The stochastics of this phase is taken to be the Markovian random-jump process. We derive explicit expressions for the atomic density matrix, the temporal photon correlations and the fluorescence spectrum. The results greatly resemble the results for a non-Gaussian diffusive phase, but deviate slightly from the corresponding expressions in the Wiener-Lévy limit of the diffusion process.

1. Introduction

In the last decade considerable effort has been made to include the effects of the finite laser linewidth in the theory of resonance fluorescence. The broadening of the single-mode laser line around the optical frequency ω_L is considered to be accomplished by a stochastic phase $\phi(t)$. The electric field at the position of the atom

$$\boldsymbol{E}(t)=E_0\,\mathrm{Re}\,\boldsymbol{\varepsilon}_L\exp[-\mathrm{i}(\omega_L t+\phi(t))] \tag{1.1}$$

gives rise to the laser profile

$$\mathrm{I}_L(\omega)=\frac{1}{\pi}\,\mathrm{Re}\int_0^\infty \exp[\mathrm{i}(\omega-\omega_L)\tau]\langle\!\langle\exp[-\mathrm{i}(\phi(t+\tau)-\phi(t))]\rangle\!\rangle\,\mathrm{d}\tau \tag{1.2}$$

where $\langle\!\langle\ldots\rangle\!\rangle$ denotes an average over the stochastic process $\phi(t)$. The stochastics of $\phi(t)$ or its derivative $\dot{\phi}(t)$ are assumed to be given, and the issue is always to average the equation of motion for the state of the atom and for the fluorescence signal over the stochastic process. Usually the phase $\phi(t)$ is taken to be the Wiener-Lévy process or equivalently, $\dot{\phi}(t)$ is taken as Gaussian white noise (Kimble and Mandel 1977, Avan and Cohen-Tannoudji 1977, Agarwal 1978, Zoller 1978). The equations occurring can then be averaged using the method of Fox (1972). A generalisation to a non-Gaussian diffusive phase has been given by Arnoldus and Nienhuis (1983a), and Dixit *et al* (1980), Zoller *et al* (1981) and Yeh and Eberly (1981) assumed the process $\dot{\phi}(t)$ to be the Ornstein-Uhlenbeck process, which has the white noise as its Lorentzian limit.

The question can be raised whether these diffusion processes properly describe the finite bandwidth of the laser and if not, how sensitive the atomic response is to the details of the stochastics of $\phi(t)$. Therefore Eberly *et al* (1984) considered the phase as a random telegraph, which can perform jumps between two values, and this model was extended by Deng and Eberly (1984) to the N-state jump process. These models, for which the phase can only assume a finite number of discrete values, are obviously not realistic. Furthermore these authors only find numerical solutions, which are hard to compare qualitatively with the diffusion models. In this paper we consider the phase as a Markov jump process, defined on the continuous range $(-\pi, \pi)$, which can be regarded as the limit $N \to \infty$ of the N-state jump process. The equations occurring are averaged exactly and explicit expressions for the state of the atom, the fluorescence spectrum and the two-photon correlation function are obtained.

2. The laser-phase stochastics

The stochastic process $\phi(t)$ is defined as a stationary Markov process on $(-\pi, \pi]$, for which the transition rate from ϕ to ϕ' is independent of the value ϕ before the transition. This random jump process is sometimes referred to as the Kubo–Anderson process (Kubo 1954, Anderson 1954). The conditional probability for $\phi(t+\tau) = \phi_2$ if $\phi(t) = \phi_1$ is then given by

$$P_\tau(\phi_2|\phi_1) = \exp(-\lambda\tau)\delta(\phi_2 - \phi_1) + [1 - \exp(-\lambda\tau)]P(\phi_2) \qquad \tau \geq 0 \tag{2.1}$$

in terms of the arbitrary probability distribution $P(\phi)$ and the transition rate λ, which gives rise to the correlation time $\lambda^{-1} > 0$. With (2.1) the laser profile is found to be

$$I_L(\omega) = |\langle\!\langle \exp(i\phi) \rangle\!\rangle|^2 \delta(\omega - \omega_L) + [1 - |\langle\!\langle \exp(i\phi) \rangle\!\rangle|^2] \frac{1}{\pi} \operatorname{Re} \frac{1}{\lambda - i(\omega - \omega_L)} \tag{2.2}$$

with the single-time average $\langle\!\langle \exp(i\phi) \rangle\!\rangle = \int d\phi\, P(\phi) \exp(i\phi)$. For a fixed value of the phase, e.g. $P(\phi) = \delta(\phi - \phi_0)$, the Lorentzian part vanishes, but in general we will have a monochromatic component of the order of $\delta(\omega - \omega_L)$ and a Lorentzian with HWHM = λ. The term $\delta(\omega - \omega_L)$ appears only if the phase has preferred values, as in the case for the random telegraph and the N-state jump process. From now on we will assume a uniform distribution

$$P(\phi) = 1/2\pi \tag{2.3}$$

which yields $\langle\!\langle \exp(i\phi) \rangle\!\rangle = 0$. Hence the laser profile is a pure Lorentzian, and the electric field (1.1) with this stochastics is known as the Lorentz wave. We remark that the assumption (2.3) is not necessary to solve the problem. All further calculations can equally well be performed for an arbitrary $P(\phi)$, which might be a discrete distribution.

The results in this paper will be compared with the corresponding expressions for the diffusive phase. The independent-increment process $\phi(t)$ with $-\infty < \phi(t) < \infty$ is defined as a Markov process with (Van Kampen 1981)

$$P(\phi, t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \exp\left(-i\rho\phi - t \int_{-\infty}^{\infty} [1 - \exp(i\rho\eta)] w(\eta)\, d\eta \right) d\rho \qquad t \geq 0 \tag{2.4}$$

where $w(\eta) \geq 0$ is the transition rate from ϕ to $\phi + \eta$. We will assume the symmetry relation $w(-\eta) = w(\eta)$. The conditional probability follows from (2.4) as

$$P_\tau(\phi_2|\phi_1) = P(\phi_2 - \phi_1, \tau) \qquad \tau \geq 0 \tag{2.5}$$

and hence the stochastics is fixed by the function $w(\eta)$, which is supposed to be given. This non-stationary process $\phi(t)$ is obviously different from the random-jump process, as follows from a comparison of (2.1) and (2.5). The laser profile however, is again a Lorentzian, where λ now follows from $w(\eta)$ by

$$\lambda = \int_{-\infty}^{\infty} (1-\cos \eta) w(\eta)\, \mathrm{d}\eta. \tag{2.6}$$

In the Gaussian limit of (2.4) we obtain

$$P(\phi, t) = (2\pi\sigma^2(t))^{-1/2} \exp(-\phi^2/2\sigma^2(t)) \tag{2.7}$$

with

$$\sigma^2(t) = t \int_{-\infty}^{\infty} \eta^2 w(\eta)\, \mathrm{d}\eta = 2\lambda t. \tag{2.8}$$

This special case is the more familiar Wiener-Lévy process.

The multiplicative stochastic differential equations have been averaged for the random-jump process by Brissaud and Frisch (1971, 1974), Shapiro and Loginov (1978) and for the independent-increment process by Arnoldus and Nienhuis (1983a, b). The correlation functions of the fluorescence field, which determine the spectral distribution and the temporal correlations of the photons, involve two-time averages. These averages factorise for the diffusion process, but for the random-jump process care should be exercised that the initial correlations are taken into account properly. We solved this problem recently (Arnoldus and Nienhuis 1986), and the results of the present paper will rely on equation (5.8) of this previous paper.

3. The state of the atom

The density operator $\rho(t)$ of a two-level atom in the strong laser field (1.1) obeys the equation

$$\mathrm{i}\hbar\, d\rho/dt = [H_{\mathrm{af}}(t), \rho] - \mathrm{i}\hbar\Gamma\rho - \mathrm{i}\hbar\Phi\rho \tag{3.1}$$

where the atomic Hamiltonian and the interaction is given by

$$H_{\mathrm{af}}(t) = -\hbar\omega_0 P_g - \tfrac{1}{2}\hbar\Omega\{d \exp[-\mathrm{i}(\omega_{\mathrm{L}} t + \phi(t))] + \mathrm{HC}\} \tag{3.2}$$

in terms of the transition frequency ω_0 between the states $|e\rangle$ and $|g\rangle$, the projector on the ground state $P_g = |g\rangle\langle g|$, the raising operator $d = |e\rangle\langle g|$ and the Rabi frequency $\Omega = E_0 \boldsymbol{\mu}_{eg} \cdot \boldsymbol{\varepsilon}_{\mathrm{L}}/\hbar$ of the dipole coupling. Spontaneous decay and collisions with perturbers are represented by the Liouville operators Γ and Φ, defined as

$$\begin{aligned} \Gamma\rho &= \tfrac{1}{2}A(P_e\rho + \rho P_e - 2d^{\dagger}\rho d) \\ \Phi &= \gamma L_g^2 - \mathrm{i}\beta L_g \end{aligned} \tag{3.3}$$

with A the Einstein coefficient, $P_e = 1 - P_g = |e\rangle\langle e|$, and γ and β the collisional width and shift. The operator L_g is defined as

$$L_g\rho = [P_g, \rho]. \tag{3.4}$$

An equivalent representation of the equation of motion follows after the transformation

$$\sigma(t) = \exp(-\mathrm{i}\omega_{\mathrm{L}} t L_g)\rho(t). \tag{3.5}$$

We then obtain

$$\mathrm{i}\, \mathrm{d}\sigma/\mathrm{d}t = \{L_{\mathrm{d}} - \mathrm{i}\Gamma - \mathrm{i}\Phi + B[\exp(-\mathrm{i}\phi(t)) - 1] + C[\exp(\mathrm{i}\phi(t)) - 1]\}\sigma \tag{3.6}$$

where we have introduced the operators B and C as

$$B\sigma = -\tfrac{1}{2}\Omega[d, \sigma] \qquad C\sigma = -\tfrac{1}{2}\Omega[d^{\dagger}, \sigma] \tag{3.7}$$

and the dressed-atom Liouvillian

$$L_{\mathrm{d}} = \Delta L_g + B + C \qquad \Delta = \omega_{\mathrm{L}} - \omega_0. \tag{3.8}$$

The operators B and C, as they appear in (3.6), account for the laser bandwidth and an average over the process $\phi(t)$ will yield the atomic state $\langle\!\langle\sigma(t)\rangle\!\rangle$.

The averages over the random-jump process are conveniently described by the Liouville operator

$$F(\omega, \phi) = \frac{\mathrm{i}}{\omega + \mathrm{i}\lambda - L_{\mathrm{d}} + \mathrm{i}\Gamma + \mathrm{i}\Phi - B[\exp(-\mathrm{i}\phi) - 1] - C[\exp(\mathrm{i}\phi) - 1]} \tag{3.9}$$

which depends parametrically on ϕ, and the single-time average

$$G(\omega) = \frac{1}{2\pi}\int_{-\pi}^{\pi} F(\omega, \phi)\,\mathrm{d}\phi. \tag{3.10}$$

If we adopt the Laplace transform

$$\tilde{\sigma}(\omega) = \int_{t_0}^{\infty} \exp[\mathrm{i}\omega(t - t_0)]\langle\!\langle\sigma(t)\rangle\!\rangle\,\mathrm{d}t \tag{3.11}$$

and assume a given non-stochastic initial state $\sigma(t_0)$, then the average of the equation of motion (3.6) becomes (Brissaud and Frish 1971, 1974)

$$\tilde{\sigma}(\omega) = \frac{1}{1 - \lambda G(\omega)} G(\omega)\sigma(t_0). \tag{3.12}$$

The Laplace inverse of this expression equals the atomic state $\langle\!\langle\sigma(t)\rangle\!\rangle$ for $t \geq t_0$, and the stationary state

$$\bar{\sigma} = \lim_{t\to\infty} \langle\!\langle\sigma(t)\rangle\!\rangle \tag{3.13}$$

is obviously the unique solution of

$$(1 - \lambda G(0))\bar{\sigma} = 0 \qquad \mathrm{Tr}\,\bar{\sigma} = 1 \qquad \bar{\sigma}^{\dagger} = \bar{\sigma}. \tag{3.14}$$

It is quite straightforward to evaluate the matrix elements of $F(\omega, \phi)$ from (3.9) with respect to the bare-state basis P_e, P_g, d, $d^{\dagger}$ of Liouville space. A subsequent average over ϕ then yields the matrix elements of $G(\omega)$, which can be applied to solve (3.14). We find the atomic steady state to be given by the steady-state matrix elements

$$\begin{aligned} n_{\mathrm{e}} &= \langle e|\bar{\sigma}|e\rangle = \frac{\tfrac{1}{2}\Omega^2(\tfrac{1}{2}A + \gamma + \lambda)}{\Omega^2(\tfrac{1}{2}A + \gamma + \lambda) + A[(\tfrac{1}{2}A + \gamma + \lambda)^2 + (\Delta - \beta)^2]} \\ \langle e|\bar{\sigma}|g\rangle &= 0. \end{aligned} \tag{3.15}$$

This population n_{e} of the excited state is identical to the result for the diffusion process. The steady-state coherence however, appears to vanish for the jump process, which was not the case for the diffusive phase. This can be understood from the fact that the coherence is proportional to the field amplitude via $\Omega\exp(-\mathrm{i}\phi(t))$, which averages out to zero for a uniformly distributed phase. The populations are determined by the field intensity Ω^2, which is independent of the phase of the field. The dependence of

n_e on the laser linewidth λ arises due to a different mechanism. From (1.1) we notice that $\dot{\phi}(t)$ effectively shifts the central frequency ω_L temporarily. The population also depends on ω_L through $\Delta = \omega_L - \omega_0$, and hence a shift of ω_L with $\dot{\phi}(t)$ alters this population. For small values of $|\Delta|$ the laser will be shifted out of resonance, which diminishes the excitation probability of $|e\rangle$, and conversely a large detuning can be reduced by $\dot{\phi}(t)$, which enhances the population of $|e\rangle$. Notice that An_e is the total number of fluorescent photons, emitted per unit time, which is also equal to the rate of photon absorption from the field. Hence n_e determines the absorption profile, as measured by the fluctuating field.

4. Photon correlations

In this section we study the time-resolved correlations between the spontaneously emitted fluorescent photons. The steady-state field intensity, expressed as the number of emitted photons per unit time, can be written as

$$I_1 = A \operatorname{Tr} R\bar{\sigma} = An_e \tag{4.1}$$

with R the photon emission operator

$$R\sigma = d^\dagger \sigma d = P_g \operatorname{Tr} P_e \sigma. \tag{4.2}$$

The I_1 is directly obtained from (3.15), and so we find that the emitted power is not affected by the change of stochastics of $\phi(t)$.

The probability density for the detection of a photon at time t and a detection at $t + \tau > t$ equals the field intensity correlation, which attains the form

$$I_2(t, t+\tau) = A^2 \lim_{t\to\infty} \langle\!\langle \operatorname{Tr} RY(t+\tau, t)R\sigma(t) \rangle\!\rangle \tag{4.3}$$

(Agarwal 1979, George 1981). This expression contains the evolution operator $Y(t+\tau, t)$ for the density operator σ, as follows from the equation of motion (3.6). The steady state is now implied by the limit $t \to \infty$, but for finite values of τ we cannot replace $\sigma(t)$ by $\bar{\sigma}$, since the average does not factorise. If we introduce

$$b(\tau) = \lim_{t\to\infty} \langle\!\langle Y(t+\tau, t)R\sigma(t) \rangle\!\rangle \tag{4.4}$$

we can write (4.3) as

$$I_2(t, t+\tau) = A^2 \operatorname{Tr} Rb(\tau). \tag{4.5}$$

The initial value of $b(\tau)$ equals

$$b(0) = R\bar{\sigma} = P_g n_e. \tag{4.6}$$

The stochastic average of (4.4) in the Laplace domain becomes

$$\tilde{b}(\omega) = \frac{1}{1 - \lambda G(\omega)} \frac{1}{2\pi} \int_{-\pi}^{\pi} \mathrm{d}\phi \, F(\omega, \phi) R \lambda F(0, \phi) \bar{\sigma}. \tag{4.7}$$

In general this cannot be written as an effective evolution operator, acting on the initial state $R\bar{\sigma}$. This is due to the finite correlation time λ^{-1} of the jump process. If we substitute the explicit forms of the operators R and $F(0, \phi)$ and of the density operator $\bar{\sigma}$, then we can prove the identity

$$R\lambda F(0, \phi)\bar{\sigma} = R\bar{\sigma}. \tag{4.8}$$

If we apply this result in (4.7), we obtain

$$\tilde{b}(\omega) = n_e \frac{1}{1-\lambda G(\omega)} G(\omega) P_g. \tag{4.9}$$

Comparison with expression (3.12) for $\tilde{\sigma}(\omega)$ shows that $b(\tau)$ equals n_e times the density operator at time τ, provided that the atom is in the ground state at time zero. This is the familiar result for the intensity correlation (Kimble and Mandel 1976, Carmichael and Walls 1976). Now we can write

$$I_2(t, t+\tau) = I_1 f(\tau) \tag{4.10}$$

where $f(\tau)$ has the significance of the conditional probability for a photon detection at time τ after a detection at time zero. The Laplace transform of $f(\tau)$ then takes the form

$$\tilde{f}(\omega) = A \operatorname{Tr} R \frac{1}{1-\lambda G(\omega)} G(\omega) P_g \tag{4.11}$$

and with the matrix calculus we obtain explicitly

$$\tilde{f}(\omega) = \frac{\mathrm{i}A}{\omega} \frac{\frac{1}{2}\Omega^2(\frac{1}{2}A+\gamma+\lambda-\mathrm{i}\omega)}{\Omega^2(\frac{1}{2}A+\gamma+\lambda-\mathrm{i}\omega)+(A-\mathrm{i}\omega)[(\frac{1}{2}A+\gamma+\lambda-\mathrm{i}\omega)^2+(\Delta-\beta)^2]}. \tag{4.12}$$

This result is again identical to the result with the diffusive phase (Arnoldus and Nienhuis 1983b). Hence the time-resolved photon correlations are the same for both models of the stochastic laser field, which is remarkable indeed.

5. The fluorescence spectrum

Before we evaluate the fluorescence spectrum for the case of excitation with a Lorentz wave, let us recall the results for the independent-increment case. The expression for the spectrum $I(\omega)$ can be extracted from a previous paper (Arnoldus and Nienhuis 1983a), if the complicated collisional effects are simplified to the impact limit (3.3). Then we find

$$\begin{aligned} I(\omega) = A n_e \operatorname{Re} \frac{1}{\pi D(\Lambda)} \bigg(& \tfrac{1}{2}\Omega^2 + (A+\lambda-\mathrm{i}\Lambda)[\tfrac{1}{2}A+\gamma+\lambda'+\mathrm{i}(\Delta-\beta-\Lambda)] \\ & + \frac{\frac{1}{2}A}{\frac{1}{2}A+\lambda+\gamma} \frac{A+\lambda-\mathrm{i}\Lambda}{\lambda-\mathrm{i}\Lambda}[\tfrac{1}{2}A+\gamma+\lambda-\mathrm{i}(\Delta-\beta)] \\ & \times[\tfrac{1}{2}A+\gamma+\lambda'+\mathrm{i}(\Delta-\beta-\Lambda)]\bigg) \end{aligned} \tag{5.1}$$

with $\Lambda = \omega - \omega_L$ and

$$\begin{aligned} D(\Lambda) = \Omega^2[\tfrac{1}{2}A+\gamma+\tfrac{1}{2}\lambda'-\mathrm{i}\Lambda) + (A+\lambda-\mathrm{i}\Lambda)[\tfrac{1}{2}A+\gamma-\mathrm{i}(\Delta-\beta+\Lambda)] \\ \times[\tfrac{1}{2}A+\gamma+\lambda'+\mathrm{i}(\Delta-\beta-\Lambda)]. \end{aligned} \tag{5.2}$$

Apart from the laser bandwidth λ, a second parameter λ' of the stochastic process appears. This parameter is defined as

$$\lambda' = \int_{-\infty}^{\infty} (1 - \cos 2\eta) w(\eta) \, d\eta \tag{5.3}$$

and comparison with the definition (2.6) of λ yields the restriction

$$0 < \lambda' \leqslant 4\lambda. \tag{5.4}$$

This λ' is an independent second parameter in general, but in the Wiener-Lévy limit (the Gaussian limit) it is related to λ as $\lambda' = 4\lambda$.

The fluorescence spectrum in the random-jump case can be derived along the very same lines as the two-photon correlation. If the driving field is stochastic with a finite correlation time, then the general expression for the steady-state fluorescence spectrum is (Arnoldus and Nienhuis 1985)

$$I(\omega) = \frac{A}{\pi} \lim_{t\to\infty} \mathrm{Re} \int_0^{\infty} \exp(\mathrm{i}\Lambda\tau) \langle\!\langle \mathrm{Tr}\, d^{\dagger} Y(t+\tau, t)(\sigma(t) d) \rangle\!\rangle \, d\tau. \tag{5.5}$$

With the abbreviation

$$a(\tau) = \lim_{t\to\infty} \langle\!\langle Y(t+\tau, t)(\sigma(t)\, d) \rangle\!\rangle \tag{5.6}$$

the spectrum (5.5) becomes

$$I(\omega) = \frac{A}{\pi} \mathrm{Re}\, \mathrm{Tr}\, d^{\dagger} \tilde{a}(\Lambda). \tag{5.7}$$

The initial value of the Liouville vector $a(\tau)$ is

$$a(0) = S\bar{\sigma} = \bar{\sigma} d \tag{5.8}$$

which defines the action of the Liouville operator S. This clearly resembles (4.6).

For the stochastics of the Lorentz wave we find

$$\tilde{a}(\Lambda) = \frac{1}{1 - \lambda G(\Lambda)} \frac{1}{2\pi} \int_{-\pi}^{\pi} d\phi F(\Lambda, \phi) S\lambda F(0, \phi)\bar{\sigma} \tag{5.9}$$

and the initial correlations become

$$S\lambda F(0, \phi)\bar{\sigma} = n_e d - \frac{\frac{1}{2}\mathrm{i}A\Omega \exp(\mathrm{i}\phi)(\frac{1}{2}A + \gamma + \lambda - \mathrm{i}(\Delta - \beta))}{\Omega^2(\frac{1}{2}A + \gamma + \lambda) + A[(\frac{1}{2}A + \gamma + \lambda)^2 + (\Delta - \beta)^2]} P_g. \tag{5.10}$$

If we set $\phi = 0$, then this is identical to the expression for $S\bar{\sigma}$ in the diffusion case. Now we can substitute (5.10) into (5.9), which yields $\tilde{a}(\Lambda)$, and with (5.7) we then obtain the spectrum $I(\omega)$. It turns out that the result is identical to the solution for the diffusive phase (5.1), provided that we take λ' equal to λ. This shows that the spectrum for excitation with the Lorentz wave is a special case of the general diffusion result, but that it differs from the Wiener-Lévy solution, since then we have $\lambda' = 4\lambda$.

The structure of the spectrum $I(\omega)$ does not depend much on the precise value of λ'. In the low-intensity limit the spectrum is independent of λ' and for high intensities the spectrum has always three separated lines (Mollow 1969). The line strengths depend on λ, but not on λ' (Arnoldus and Nienhuis 1983a). Only the lineshapes are sensitive to the value of λ'. This is illustrated in figure 1. Also in the limit of a large laser linewidth λ, the λ' dependence vanishes in general. Thus it turns out that even

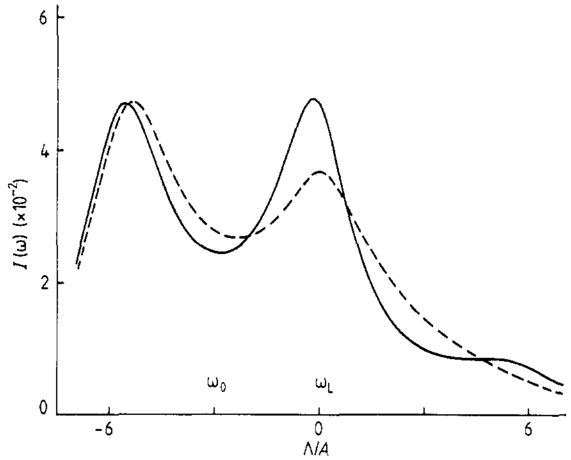

Figure 1. Plot of the fluorescence spectrum $I(\omega)$ from (5.1) as a function of $\Lambda = \omega - \omega_L$, with $\Omega = 5A$, $\Delta = 3A$, $\gamma = \frac{1}{3}A$, $\beta = 0$ and $\lambda = A$. The full curve corresponds to random jumps in the laser phase, so $\lambda' = \lambda$, and the broken curve is the spectrum for a Gaussian diffusive phase ($\lambda' = 4\lambda$). The peak on the right-hand side at ω_L is the elastic Rayleigh line. Its shape depends on λ', but the integrated strength is independent of λ'. The line near the atomic resonance ω_0 is the laser linewidth induced fluorescence line, which vanishes for $\lambda = 0$. (Recall that for $\lambda = 0$ the spectrum is symmetric around $\omega = \omega_L$.) The shape of this phase fluctuation induced line is almost unaffected by the change of λ'. The small slope in the right wing of the Rayleigh line is the appearance of the three-photon line, which is strongly suppressed by the phase fluctuations.

with a spectral resolution, the atomic response is only slightly modified by this change of the stochastics of the laser field.

6. Conclusions

We investigated the scattering of photons from a stochastically fluctuating laser field, the Lorentz wave, by a two-level atom in the presence of collisions. The laser phase is assumed to perform random jumps at random instants, and the probability distribution is taken to be uniform (and therefore time independent). The laser profile is then a Lorentzian. Closed expressions were obtained for the atomic density matrix, the photon correlation function and the fluorescence spectrum. The results were compared with the corresponding expressions for the case of a diffusive phase, which gives rise to the same Lorentzian laser profile. The stochastics of a diffusive phase is essentially different from the stochastics of a random-jump process, but it turned out that the observable properties of the fluorescence signal are indistinguishable. Even a perfect frequency resolution in the detection of the fluorescent photons does not discriminate between both driving fields. The atomic response to a Lorentz wave is a special case of its response to a laser field with a non-Gaussian diffusion phase.

The structure of the formulae for the stochastic averages over the random jump process is very different and much more involved than the simple expressions for the diffusion process. The question should be raised whether this correspondence in the final results is merely a coincidence, or whether it could be expected. Both the random jump process and the independent-increment process are Markov processes, which are determined by the transition rate $W(\phi'|\phi)$. For random jumps on $(-\pi, \pi]$ with a uniform distribution, this rate equals $\lambda/2\pi$, independent of ϕ' and ϕ. For the diffusion process we have $W(\phi'|\phi) = w(\phi' - \phi)$. If we choose this rate also independent of $\phi' - \phi$ (apart from the slight difficulty with its normalisation on $(-\infty, \infty)$) and proportional to λ, then the stochastics of both processes are essentially identical. For $w(\eta)$ independent of η, we find from (2.6) and (5.3) that λ' equals λ indeed.

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